Passive vibration isolation using a Roberts linkage

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The article presents a new ultra-low frequency passive vibration isolation device as part of the pre-isolation stage for AIGO (Australian International Gravitational Observatory). A description of this device together with a theoretical model and isolation performances are presented. With the current set-up, we obtained 32dB isolation at 1 Hz.

I. Introduction.

Since the first interferometric gravitational wave detectors were developed, the main difficulty in attaining high strain sensitivity has been in reducing the various technical noise sources in the interferometer. The seismic motion of the test masses in such an interferometric antenna is the first noise source that requires attention. Efforts to improve isolation against seismic motion have resulted in several proposed very-low frequency passive isolators, such as the X-pendulum[1], the folded pendulum (Watt's linkage)[2] and the Scott-Russel linkage[3]. Of these, the last two produced 90-100 dB isolation in a single stage device at respectively ~10 Hz and 2 Hz.

The Roberts linkage[4] is widely known as a structure that achieves near straight-line motion from rigid links and pivots and as such it provides another means to simulate the motion of a long radius pendulum in a relatively short height. It may be readily generalized to cylindrical symmetry to give *x*-*y* motion and was first proposed for ultra-low frequency pre-isolation by J. Winterflood[5]. While apparently

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not well known, and not recognized as a Roberts linkage, the structure has been employed for horizontal isolation previously[6].

We have built and used two small ultra-low frequency seismometer devices using this structure[7], and have now constructed a large model capable of suspending several hundred kilograms as part of an ultra-low frequency pre-isolation structure for ACIGA's (Australian Consortium for Interferometric Gravitational Astronomy) high optical power interferometer project.

In this paper we present the first performance measurements on this Roberts linkage isolator. We have designed the device to be the second horizontal stage of a very low frequency tilt-rigid three-dimensional pre-isolator[8], for which all stages have resonance frequencies below 100 mHz. Here we concentrate on the Roberts linkage while in a forthcoming paper we will present the entire pre-isolator.

The purpose of a pre-isolator is to reduce the residual seismic motion to well below the amplitude of the ocean wave driven microseismic peak. In conventional vibration isolators the worst residual motion normally occurs near 0.5 Hz to 1 Hz due to the normal mode peaks of multiple pendulum stages. The residual motion causes many difficulties in optical cavity lock acquisition since the motion usually exceeds many optical wavelengths. Pre-isolation structures can reduce the seismic drive to the pendulum modes, in principle allowing the residual motion to be reduced to the nanometre level[9]. In addition, a pre-isolator is a useful device to allow smooth translation of the entire isolation chain together with the test mass by useful distances (e.g. millimetres). This can be used to set optical cavity length and to counteract any thermal expansion and earth tide effects.

Given that exceptional results were obtained with a Scott-Russel linkage[3] in our laboratory, it is worth emphasising the reason for now choosing the Roberts linkage. The main reason for using a Roberts linkage instead of a Scott-Russel is its simplicity and easy construction as a two-dimensional pendulum. A second reason for this option is that because of the geometrical shape (a parallelepiped), the space inside it can be used in attaching the other stages of isolation without changing the height of the isolation chain. Thus an additional stage of isolation can be accommodated without any extra space. In this paper we will first introduce the Roberts linkage geometry. Then we will present a theoretical model for the linkage and the transfer function set- up. Finally, we present experimental results and compare them with the theoretical predictions.

II. The Device.

As is the case with devices such as the Watt's linkage and Scott-Russel linkage[5], the Roberts linkage pre-isolator illustrated in Fig. 1 works by causing the mass suspension point P to move in an almost flat horizontal plane, so that the suspended mass gravitational potential energy is almost independent of displacement. Small modifications in the geometry allow point P to move in very shallow arc, which defines the effective length of an equivalent pendulum for the Roberts linkage. A one-dimensional Roberts linkage with a suspended mass is shown in Fig. 1(*a*). It consists of a rigid frame attached to two wires. This linkage can be imagined as a **W** shape, where the sidelines represent the two suspension wires and the central inverted **V** is the rigid frame. If we take a copy of the one-dimensional linkage and rotate it 90° around a vertical axis with an appropriate rigid central section, a two-dimensional Roberts linkage[5] is obtained as shown in Fig. 1(*b*). This structure is over-constrained but intrinsic material flexibility and simple wire length adjustment make it a practical solution. The over-constraint can be removed by going to a three-wire design as we tested for the small seismometer[7], but for installation in our cubic shaped pre-isolator, a square structure was greatly preferred.

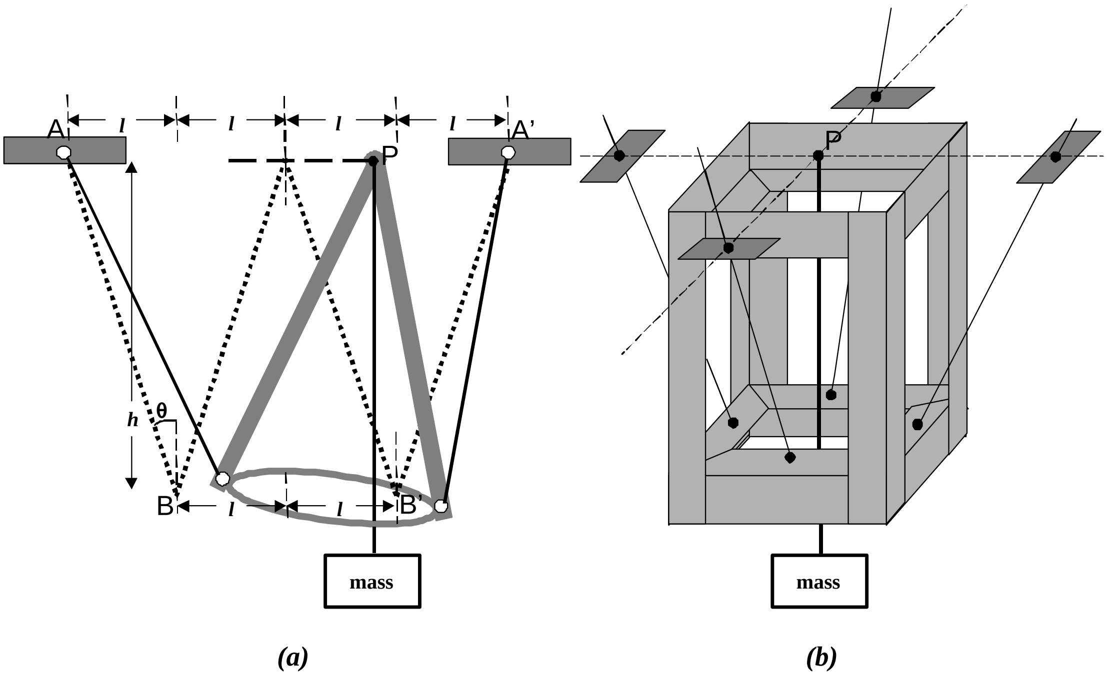

FIG. 1. The Roberts linkage diagram: (*a*) – the one-dimensional Roberts linkage with a suspended load, (*b*) – the two-dimensional Roberts linkage with a suspended load.

The classical Roberts linkage[4] requires distances AB, BP, PB' and A'B' to be equal and the distance AA' to be twice the distance BB' as shown in Fig. 1(*a*). The two-dimensional device illustrated is constructed of 60mm x 60mm x 6mm aluminium angle stock with a height *h* = 692mm, from the upper to the lower suspension point. The distance *l*, as shown in Fig. 1(*a*) is equal to 273 mm. This gives a value of 21.5 degrees of arc for the angle θ, between the vertical axis and the suspension wire of the linkage.

The four suspension wires are anchored to the structure at the bottom, and are supported by an adjustable threaded housing at the top, allowing the tension in the suspension wires to be matched and the linkage to be accurately positioned and levelled. The other stages of the suspension chain designed at AIGO[10] will be suspended from the Roberts linkage, at the point P indicated in Fig. 1. There is extra weight on top of the frame, such that the position of the Roberts linkage center of mass is near the suspension point of the mass load, to minimise the sensitivity of resonance frequency to the suspended load.

III. Theoretical model.

In this section we review the transfer function of a simple pendulum, and then generalise it to a physical pendulum.

The transfer function and the phase angle for a simple pendulum have the following expressions:

$$\left|\frac{X_{output}}{X_{input}}\right| = \left(\frac{1+Q^{-2}}{\left(1-(\omega/\omega_{res})^2\right)^2+Q^{-2}}\right)^{1/2}, \tag{1}$$

$$\alpha = arctan\left(\frac{-(\omega/\omega_{res})^2}{Q\left(1-(\omega/\omega_{res})^2+Q^{-2}\right)}\right), \tag{2}$$

where ω is the excitation frequency, ω_{res} is the resonance frequency, and the quality factor is given by $Q = 1/\varphi_{Pend}$. The damping factor of the pendulum has the following expression $\varphi_{Pend} = \varphi\,(k_g+k_s)/k_s$ and the spring constant of the pendulum is given by $k_{Pend} = k_g+k_s' = (k_g+k_s(1+i\varphi)) = (k_g+k_s)(1+i\varphi_{Pend}) = k(1+ i\varphi_{Pend})$, where k_g is the gravitational spring constant and k_s' is the flexure spring constant.

Since the Roberts linkage utilises a distributed mass, it is necessary to model it as an equivalent compound pendulum. For a physical pendulum (see Fig. 2) the transfer function at any point P on the line joining the suspension point with the CM and the phase angle have the following expressions:

$$\left|\frac{X_P}{X_O}\right| = \left(\frac{\left(1-\boldsymbol{A}(\omega/\boldsymbol{\omega_{res}})^2\right)^2+Q^{-2}}{\left(1-(\omega/\boldsymbol{\omega_{res}})^2\right)^2+Q^{-2}}\right)^{1/2}, \tag{3}$$

$$\alpha' = arctan\left(\frac{-(1-\boldsymbol{A})(\omega/\omega_{res})^2}{Q\left(\left(1-\boldsymbol{A}(\omega/\omega_{res})^2\right)\left(1-(\omega/\omega_{res})^2\right)+Q^{-2}\right)}\right), \tag{4}$$

where

$$\boldsymbol{A} = 1-\frac{ml_{CM}\,l_{OP}}{I_O} = 1-\frac{ml_{CM}\,l_{OP}}{I_{CM}+ml_{CM}^2}, \tag{5}$$

is a constant that depends on the suspension point moment of inertia (I_O), on the distance from the suspension point to the CM (l_{CM}) and to the measurement point ($l_{OP} = l_{CM} + l_P$) and on the mass of the system.

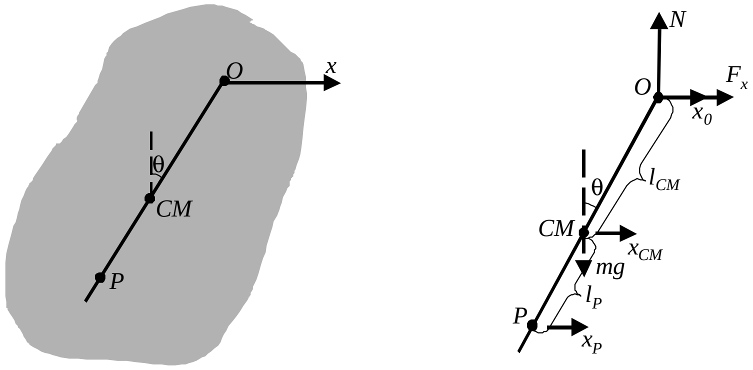

FIG. 2. Arbitrary physical pendulum.

It can be seen that for a simple pendulum the CM transfer function tends to zero at high frequencies, but for a compound pendulum, the transfer function tends to a finite value:

$$\left|\frac{X_{CM}}{X_O}\right| \rightarrow \boldsymbol{A} = 1 - \frac{ml_{CM}^2}{I_O} = 1 - \frac{ml_{CM}^2}{I_{CM} + ml_{CM}^2} = \frac{I_{CM}}{I_{CM} + ml_{CM}^2} = \frac{r_g^2}{r_g^2 + l_{CM}^2}, \tag{6}$$

where r_g is the radius of gyration.

If $\boldsymbol{A} = 0$, the physical pendulum transfer function turns into the simple pendulum transfer function [eq. (3) → eq. (1)]. At this point, called the center of percussion (CP) $l_{OP(A=0)} = \frac{I_{CM} + ml_{CM}^2}{ml_{CM}} = l_{CM} + \frac{r_g^2}{l_{CM}}$ and the compound pendulum behaves as a simple pendulum having this equivalent length.

IV. Measurement set-up and results.

A schematic diagram of the experimental set-up is presented in Fig. 3, where the position of the Roberts linkage in the pre-isolator is indicated. The device is attached to the first three-dimensional pre-isolator stage[8], which consists of a vertical LaCoste stage and a horizontal inverse pendulum. Four high tensile steel wires of 1.4 mm diameter were used to suspend the Roberts linkage from the inverse pendulum. For these measurements the inverse pendulum was not optimally tuned but was used as a tilt-rigid shake-testing frame. Its resonance frequency was 780 mHz. The Roberts linkage was carefully balanced and levelled to achieve a resonance period of 20 sec.

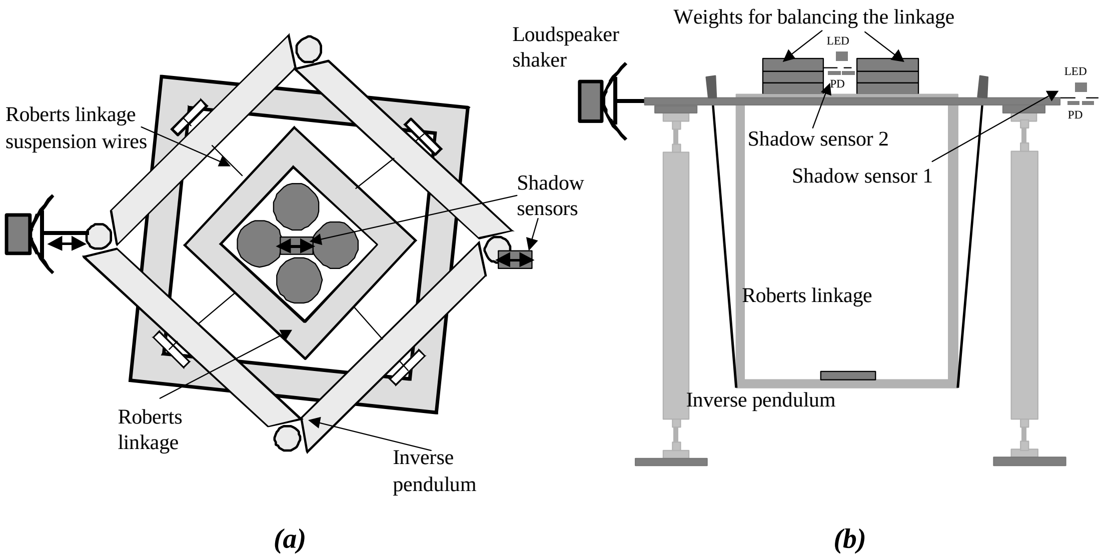

FIG. 3. Roberts linkage as part of the pre-isolator and measurement set-up diagram: **(*a*)** – top view; **(*b*)** – lateral view. A loudspeaker is used to shake the pre-isolator horizontally. The weights on top and bottom of the Roberts linkage are used to tune the resonance frequency of this device. The one-dimensional shadow sensors are placed as indicated and the measured transfer function is taken as the signal from shadow sensor 2 over the signal from shadow sensor 1. This drawing is not at scale.

A mass of 60 kg was distributed on top of the Roberts linkage at a height of 140 mm above the suspension point level. Another 6 kg were arranged on the lower part of the linkage frame to make the tuning easier. The total mass of the system was such that the centre of mass was very close to the height of the suspension point. A swept sine transfer function was recorded. To measure the transfer function shadow sensors were located as shown in Fig. 3, allowing measurement of the displacement of the first pre-isolator stage and the displacement of the Roberts linkage with respect to the ground.

The excitation signal was applied with a loudspeaker calibrated to provide a force of 6 N/A. This signal was applied at approximately 45° with respect to the two eigen-mode directions of oscillation and the output was measured in the same direction. In order to keep a good signal to noise ratio, the measurement was made over three different frequency ranges with appropriate source amplitudes (2 V amplitude for 20 mHz-400 mHz, 1 V for 350 mHz-1 Hz, and 3 V for 1 Hz-10 Hz). For the experiments reported here differences between the periods of oscillation in the two orthogonal directions were encountered, such that one had 18.5 sec period and the other had 25.5 sec period. The reason for these differences can be assigned to small differences in distances between suspension points of the Roberts linkage in the two orthogonal directions as well as to imperfections in the suspension wire cylindricity and clamping isotropy. These last two factors could be improved by lapping the wires to match the collets as described in[11].

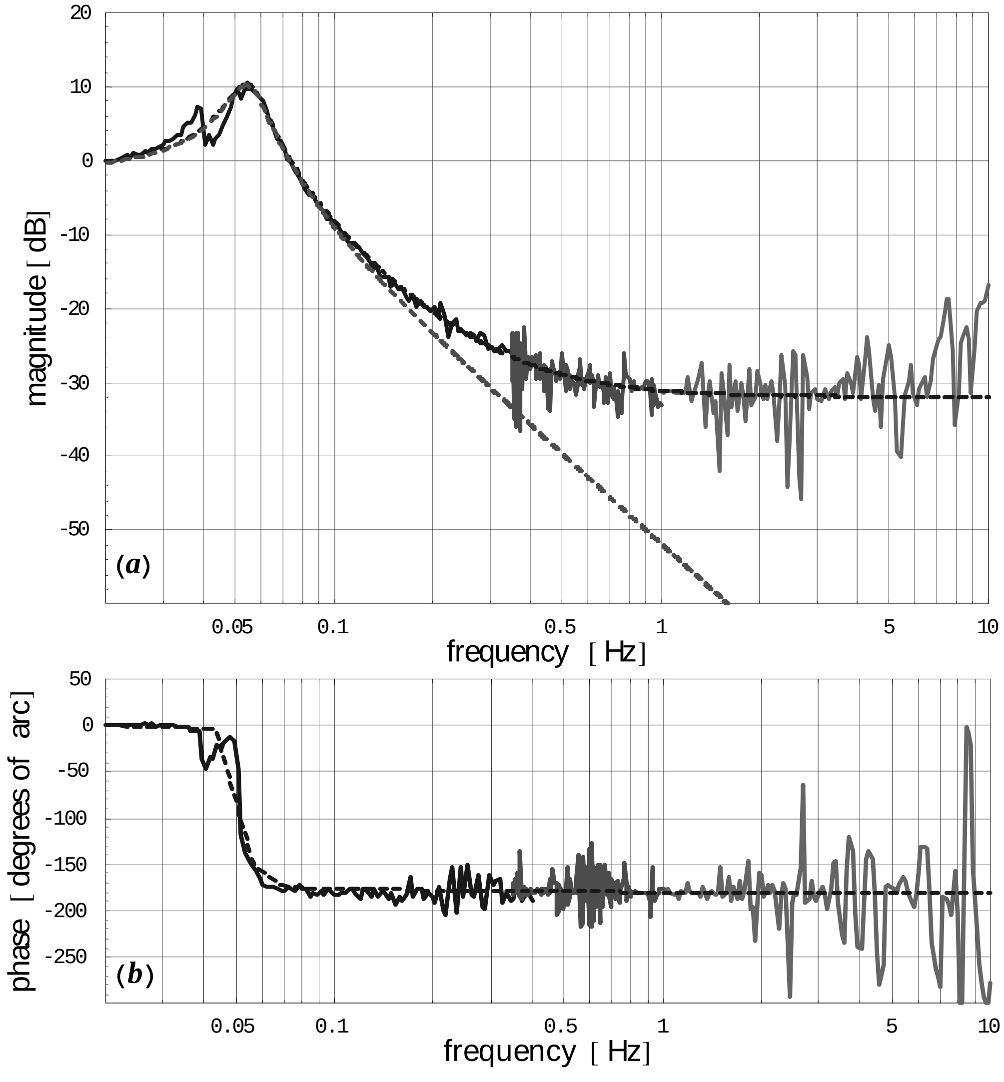

FIG. 4. (*a*) – Roberts linkage transfer function. The two dashed lines represent physical pendulum model transfer functions for ***A*** = 0 (at the CP) and for ***A*** = -0.03, the last one fitting our experimental data. **(*b*) –** Phase plots. The dashed line shows theoretical model prediction (with ***A*** = -0.03).

Cross coupling between these two modes leads to a double peak response in the transfer function as is seen in Fig. 4. The main peak at 54 mHz shows the resonance frequency of the Roberts linkage approximately in the direction of measurement and the other small peak at 39 mHz is due to cross coupling with the perpendicular direction of oscillation. This arises because the excited direction is not exactly in line with a modal axis. The two dashed lines in Fig. 4(*a*) show theoretical transfer functions for the CP ($\boldsymbol{A}$ = 0) and for $\boldsymbol{A}$ = -0.03. We also present the phase plots, for the measured data and for the physical pendulum model (dashed line) in Fig. 4(*b*). We used a value of 3.7 for the quality factor in this model.

Fig. 4 shows the isolation of the Roberts linkage to be that expected for a physical pendulum when the constant $\boldsymbol{A}$ has a value of –0.03, yielding a maximum isolation of –32 dB. For use as a pre-isolator this is an adequate level of isolation.

Better isolation can be achieved for frequencies greater than 0.2 Hz if we suspend the isolation chain from the CP of the Roberts linkage. However, suspending the load at the CP will make the tuning and stability of the Roberts linkage dependent on loading and is therefore not desirable. An optimal design, which achieves both low frequency tuning and high frequency isolation, places both the CP and the CM at the desired suspension point.

As normally occurs with low frequency structures, the Q-factor is low because the negative spring created by the geometry cancels the real spring constant but leaves the dissipative terms unchanged.

V. Discussion.

A novel very-low frequency horizontal vibration isolation device based on the Roberts linkage has been presented. The device has advantages in simplicity of design and economical usage of the space. The simplicity is also reflected in ease of set-up. As long as the device is adequately loaded, periods of up to 40 seconds are readily achieved. With a resonance period of 18.5 sec, an isolation of 32 dB at 1 Hz was obtained. This is sufficient to significantly attenuate the seismic excitation of the normal modes of an isolation chain. The 32 dB isolation floor is due to the distributed nature of the Roberts linkage mass, which prevents the CP from being located at the CM. A better isolation could be achieved by reducing the mass of the Roberts linkage frame compared to its mass load or by having the isolation chain attached to the CP of the Roberts linkage. An isolation improvement of about 20 dB at 1 Hz should then be achieved. We point out however, that suspending the following isolation stages from the CP point makes the Roberts stage sensitive to the total mass load below it. Since it is impossible with a rigid structure to locate the CP at the same point as the CM, a possible solution may be to add one or more load pendulum masses, which can change the CM (at ultra-low resonance frequency) without affecting the high frequency dynamic response and the CP.

Because of imperfections in construction the two orthogonal directions of oscillation of the Roberts linkage typically had five seconds difference in period. This shows up as a kink in the transfer function at 40 mHz, but has little effect above resonance. We chose to build a rectangular design for reasons of space and simplicity, but a three-wire device would reduce the number of factors contributing to this anisotropy and may have an advantage in some cases. Low frequency devices such as the Roberts linkage are always susceptible to drifts. Hence, we always choose to control the working point with a very low frequency DC position control.

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