Sparse Sky Grid for Coherent network analysis

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with

Olivier Rabaste and Éric Chassande Mottin, APC, Paris
(arXiv:0905.4832)

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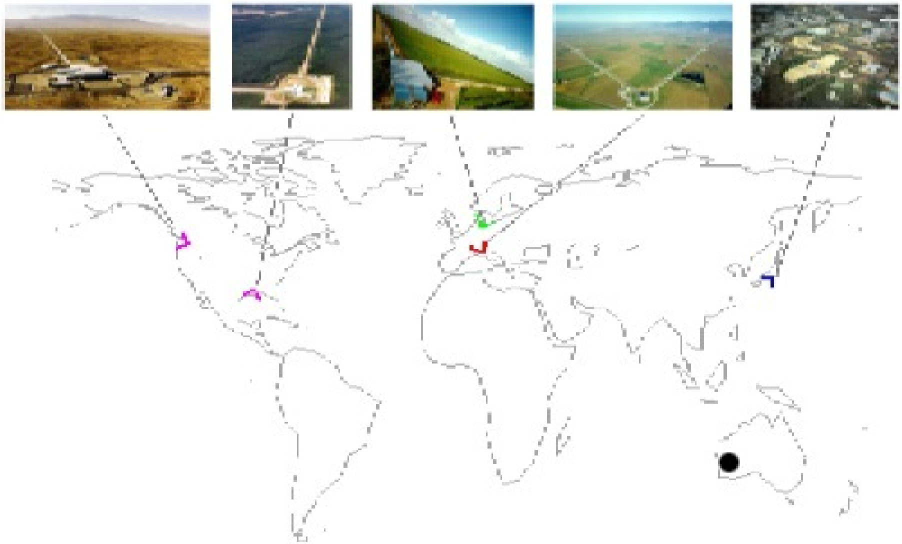
Network Analysis – Sky Sampling Problem



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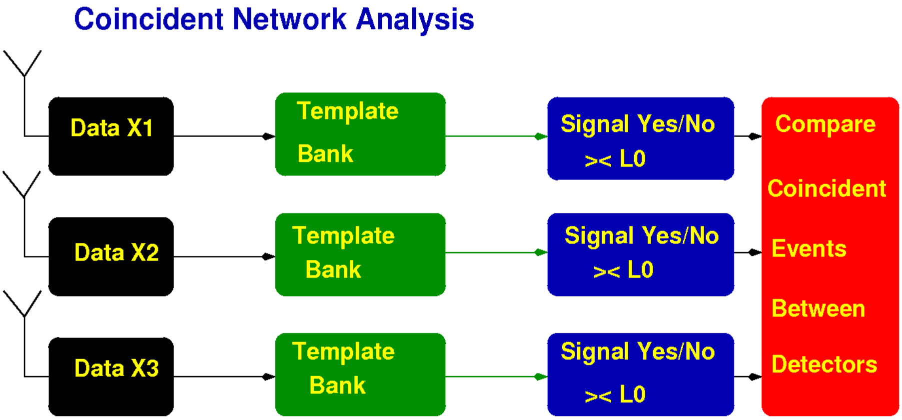

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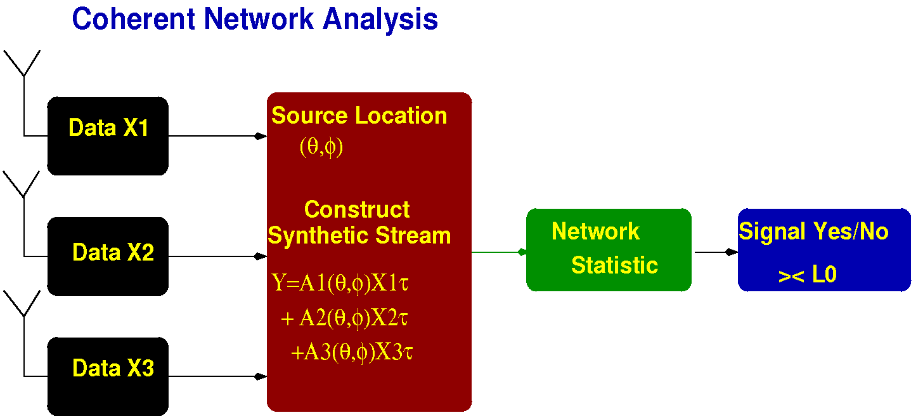

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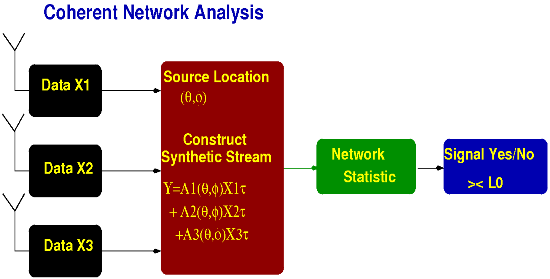
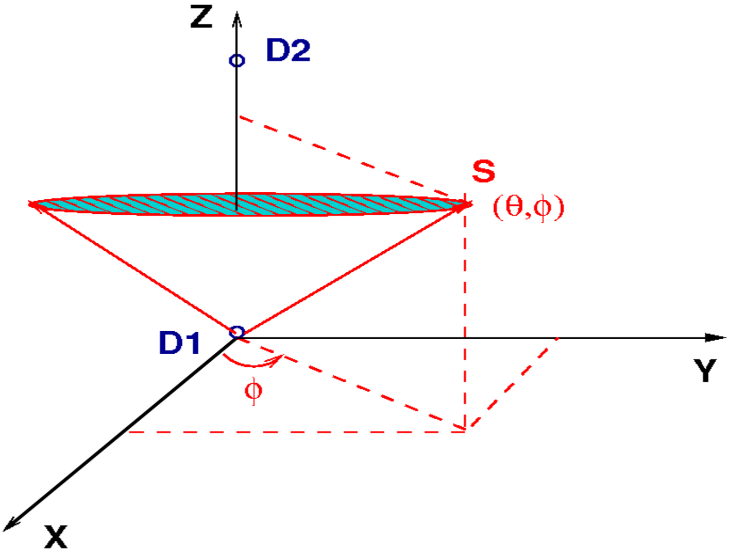


How to sample the sky ?

General Practice: Uniform Sky-grid

Intuition: Sky-grid tuned to signal and network geometry

Two Detector Network



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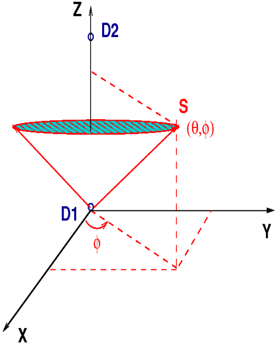


Light travel time between D1-D2 ;

$$T2 = ||D1 - D2||/c$$

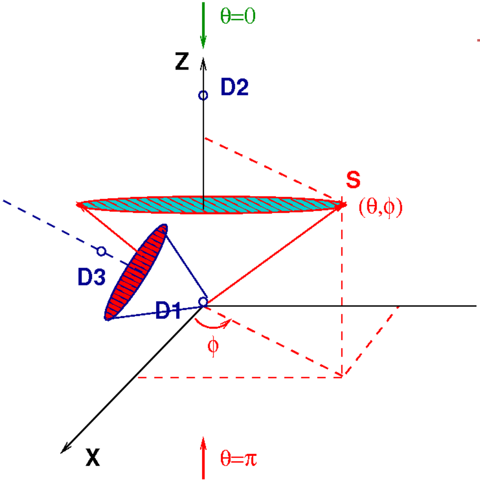
Wave arrival time delay at D2 w.r.t. D1

$$\tau_2 = T2\ cos(\theta)$$

$$-T2 \leq \tau_2 \leq T2$$

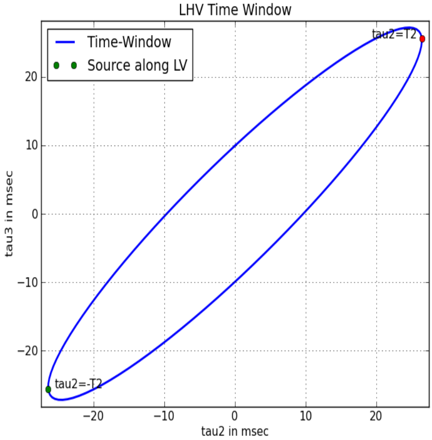
Two detectors network;
Determine θ
Degeneracy in ϕ.

Three Detector Nework: (τ_2, τ_3)



$$\tau^{\mathbf{T}} \mathbf{A} \tau \leq \mathbf{B} \qquad \tau^T = [\tau_2 \quad \tau_3], \; A_{2\times 2}$$

Time window == Ellipse in (τ_2, τ_3)



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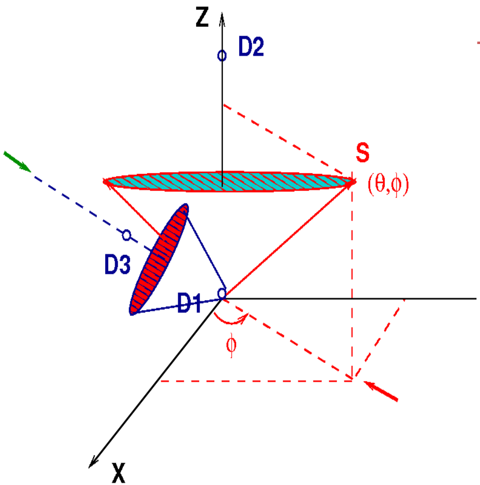

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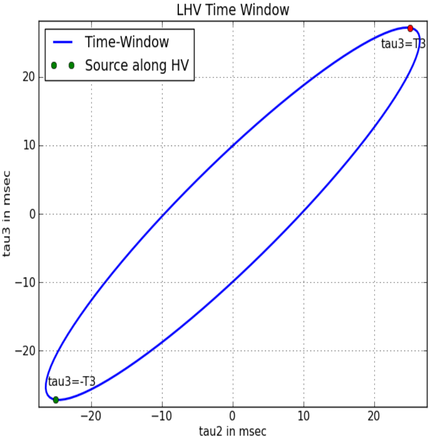

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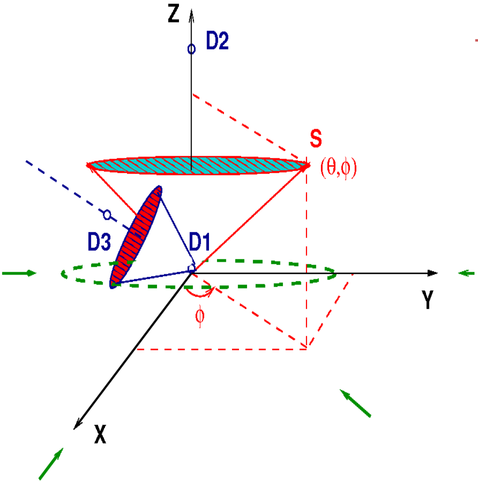

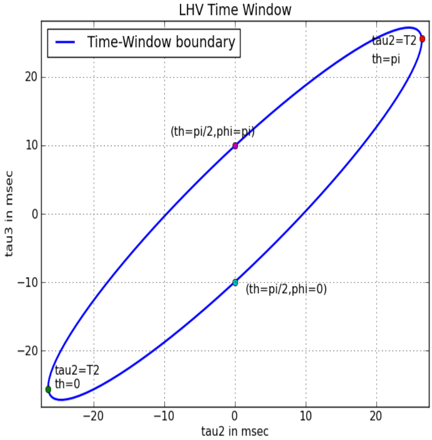

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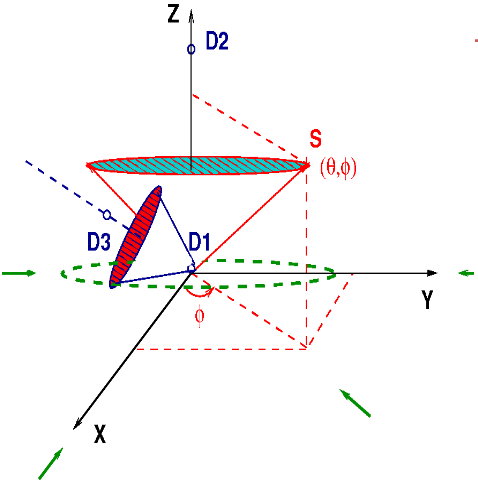

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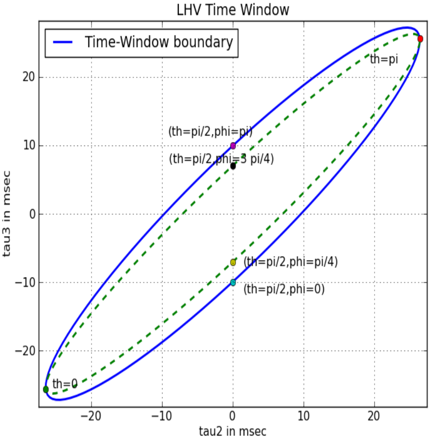

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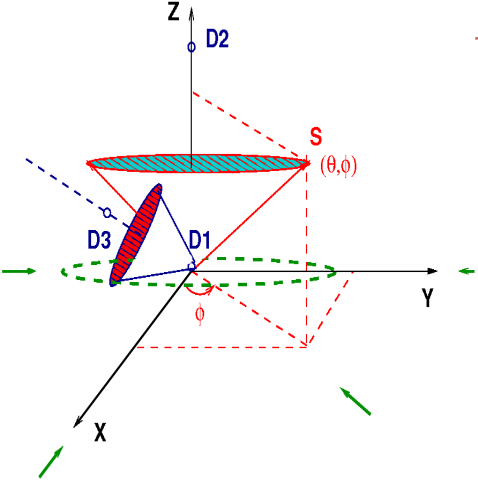

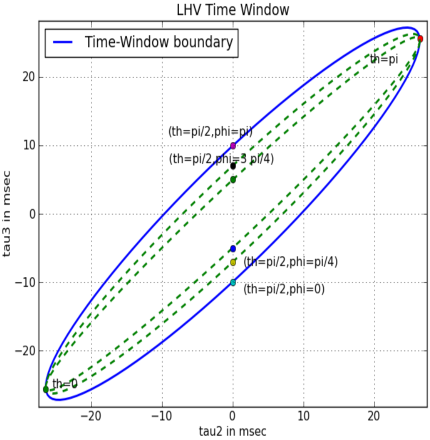

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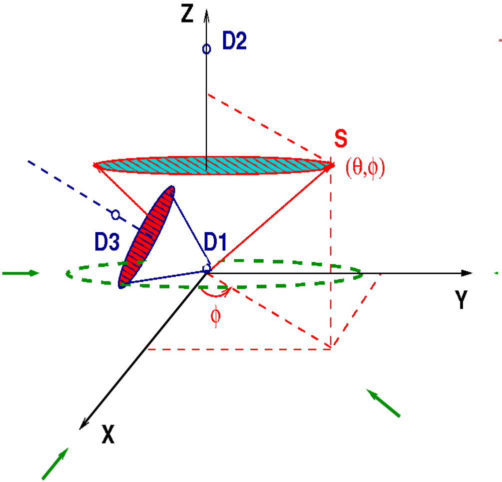

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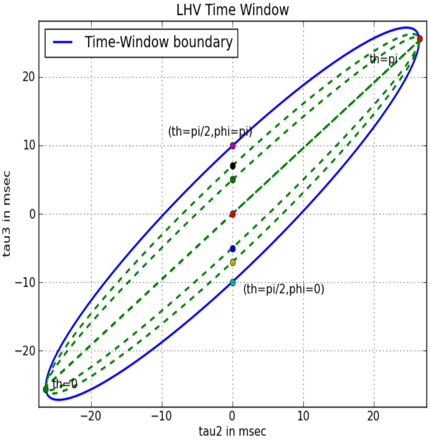

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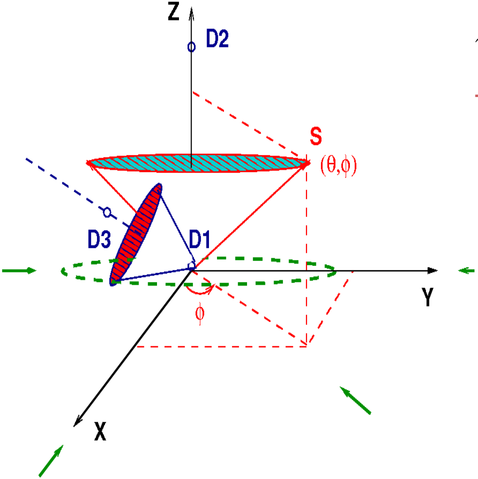

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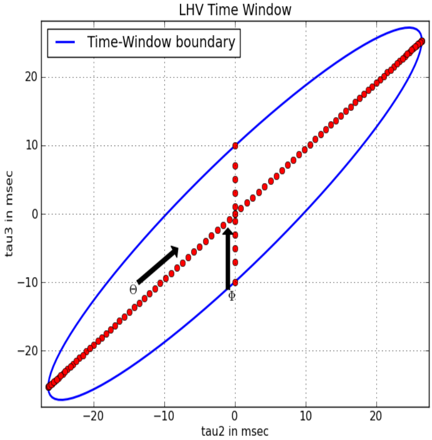
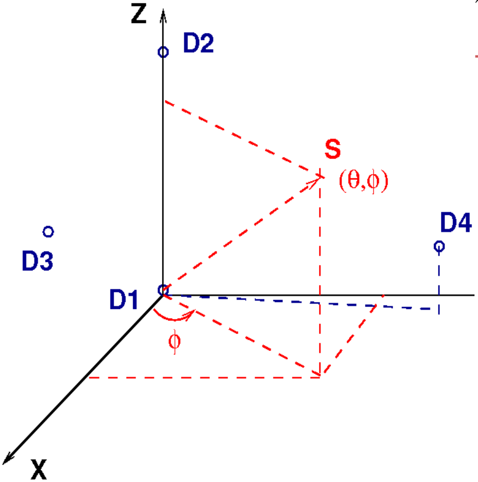


$(\theta, \pm\phi) \Rightarrow (\theta, \phi)$ – Degeneracy in ϕ

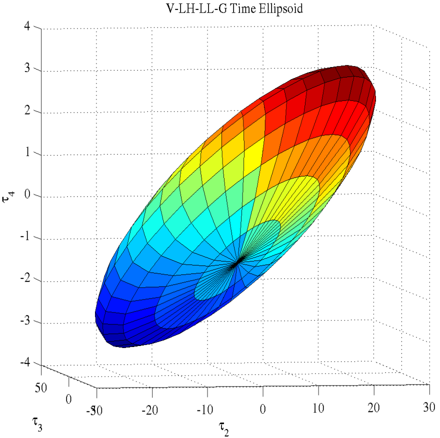
Source above and below D1-D2-D3 plane maps to same (τ_2, τ_3)*point*

Four Detector Network: (τ_2, τ_3, τ_4)



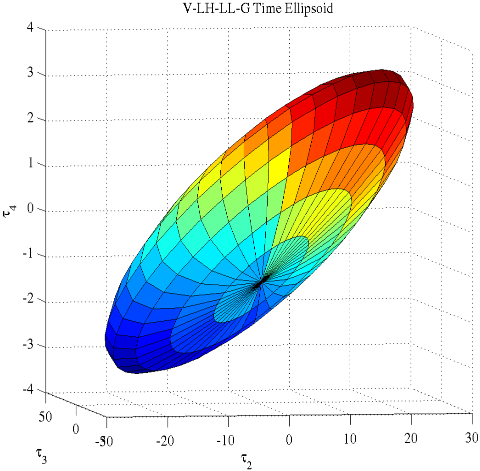
$$\tau^{\mathbf{T}} \mathbf{A} \tau = \mathbf{B} \qquad \tau^{\mathbf{T}} = [\tau_2 \;\; \tau_3 \;\; \tau_4], \quad \mathbf{A}_{3\times 3}$$

Time window: Ellipsoid in (τ_2, τ_3, τ_4)

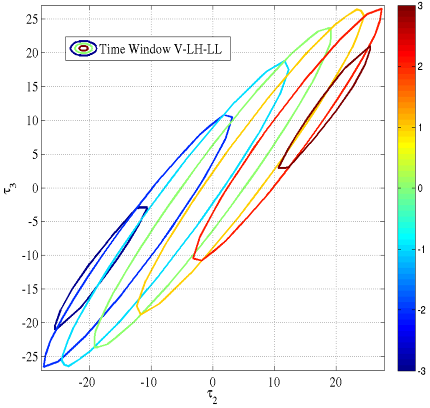


Four Detector Network: (τ_2, τ_3, τ_4)

Time window: Ellipsoid in (τ_2, τ_3, τ_4)

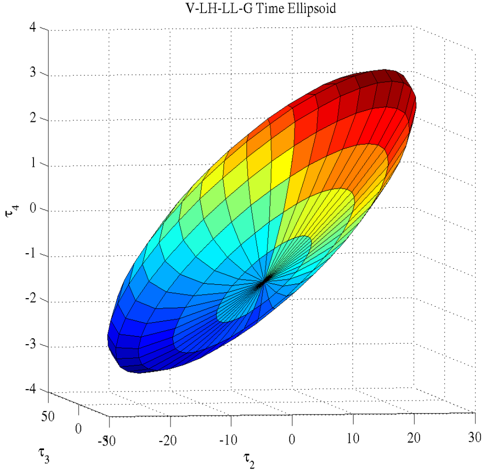


Projection on $\tau_2 - \tau_3$ plane:

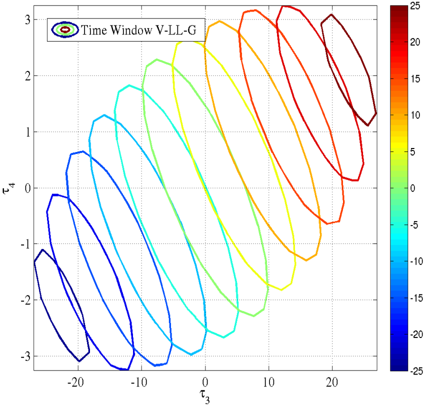


Four Detector Network: (τ_2, τ_3, τ_4)

Time window: Ellipsoid in (τ_2, τ_3, τ_4)

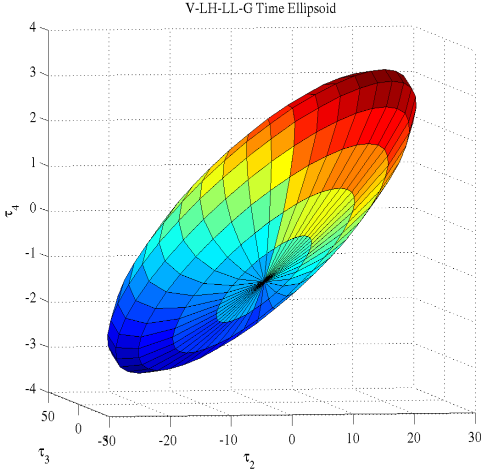


Projection on $\tau_3 - \tau_4$ plane:

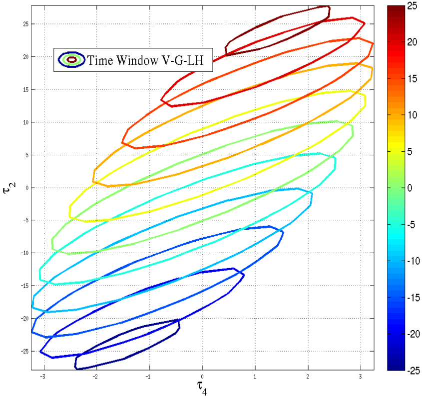


Four Detector Network: (τ_2, τ_3, τ_4)

Time window: Ellipsoid in (τ_2, τ_3, τ_4)

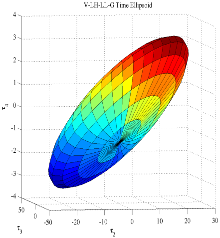


Projection on $\tau_4 - \tau_2$ plane:



Four Detector Network: (τ_2, τ_3, τ_4)

Time window: Ellipsoid in (τ_2, τ_3, τ_4)



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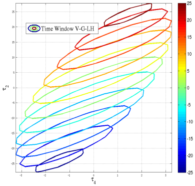


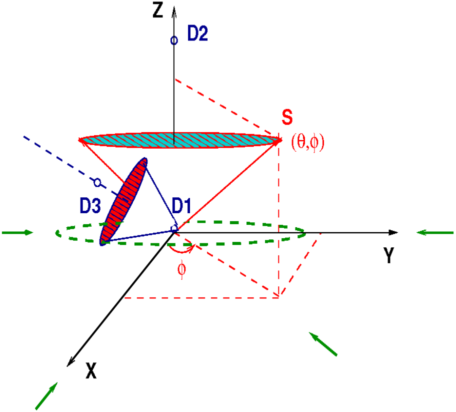
Four detectors can determine source location (θ, ϕ)

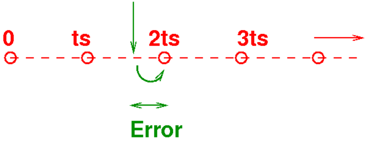
D detector network $(D \geq 4)$

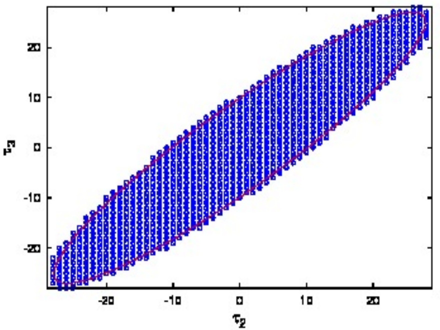
Number of ellipsoids: ${}^{D-1}C_3$

Each time-delay triplet gives independent estimate of (θ, ϕ).

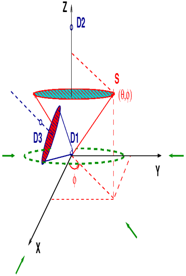
Time-delay window and the sky grid

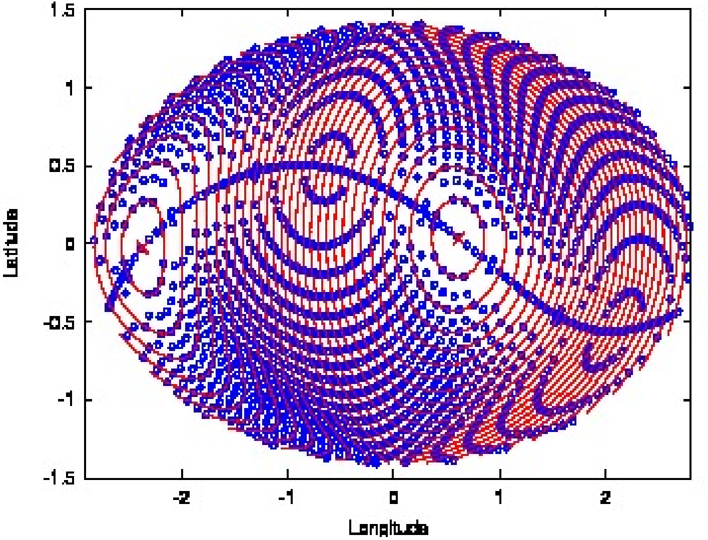




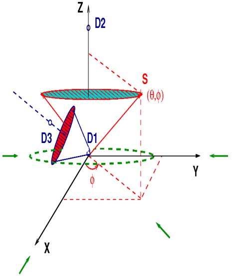


Time-delay window and the sky grid





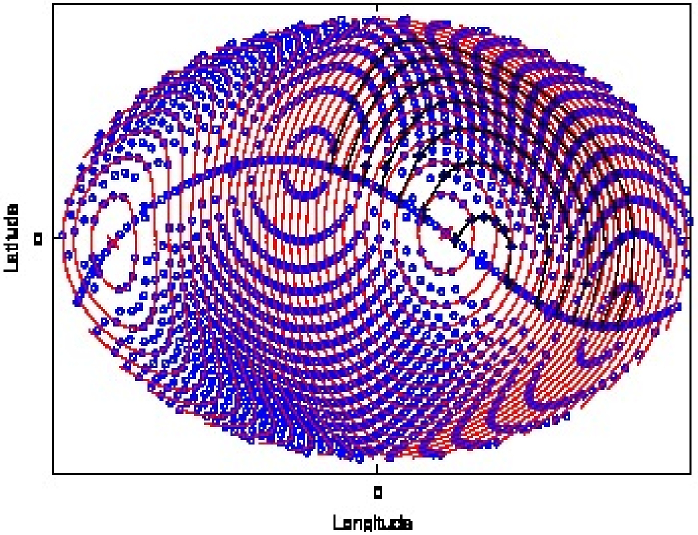
Time-delay window and the sky grid



Red: τ_2 Contour

Black: τ_3 Contour

Blue: $\tau_2 = \tau_3 + K$

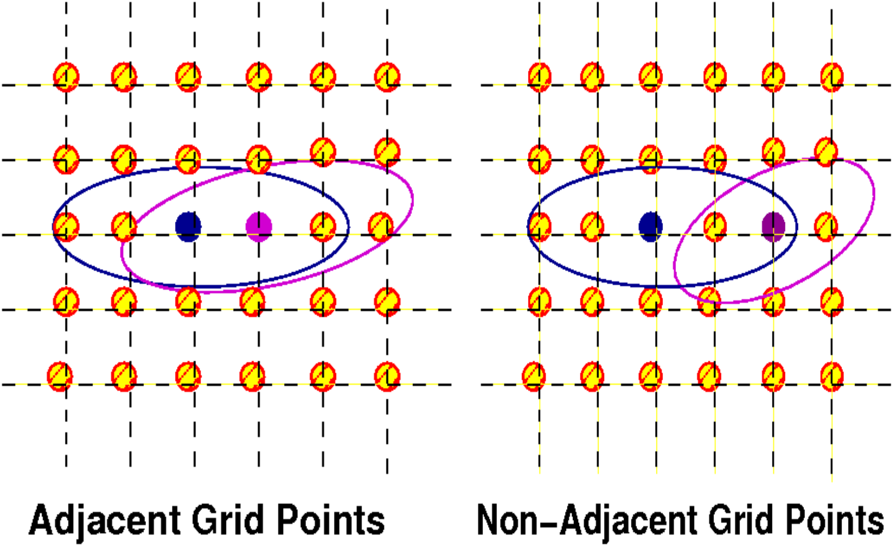


Sparse sky grid:

Do we need all the sky-grid points?

Tolerable SNR loss $\Rightarrow$ Small ellipse around (θ_0, ϕ_0)
No need to sample finer than this elliptical bin

Sparse sky grid:



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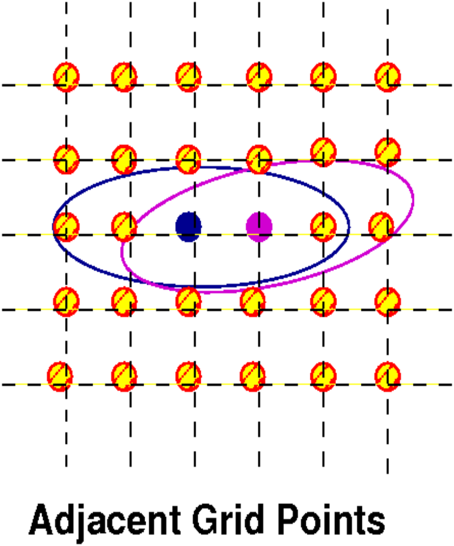
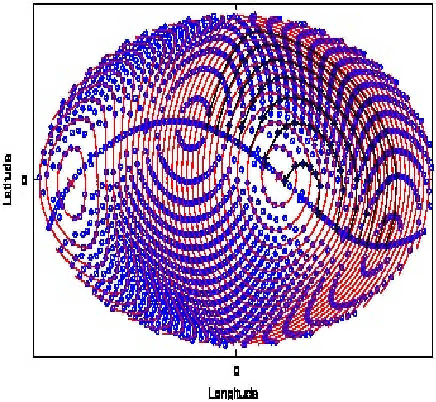


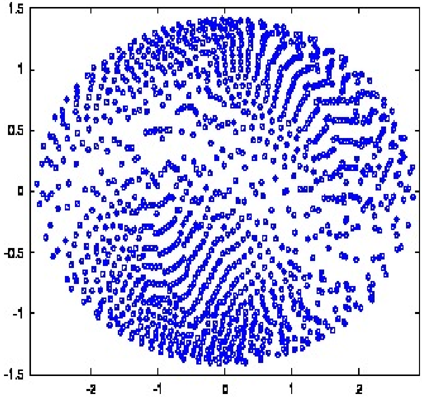
Find sparser set:
Each sample point in the original set is adjacent to at least one point in sparser set.

Map to the Set Covering Problem
Solve with the Greedy algorithm

Sparse sky grid:

Three detector case





Exhaustive Grid
$\sim$ 28000

Sparse Grid
$\sim$ 1200

Sky Sampling Features:

- Sparse grid uses the network geometry and the inout GW signal.
- Get an exhaustive set and using the greedy algorithm, obtain a sparse set
- For sky sampling, it becomes more efficient. Due to the sphere geometry, template placement could be more taxing.
- Adjacency condition takes care of the tight spacing. There is a scope of improving the condition if required.
- Performs as good as the exhaustive set.

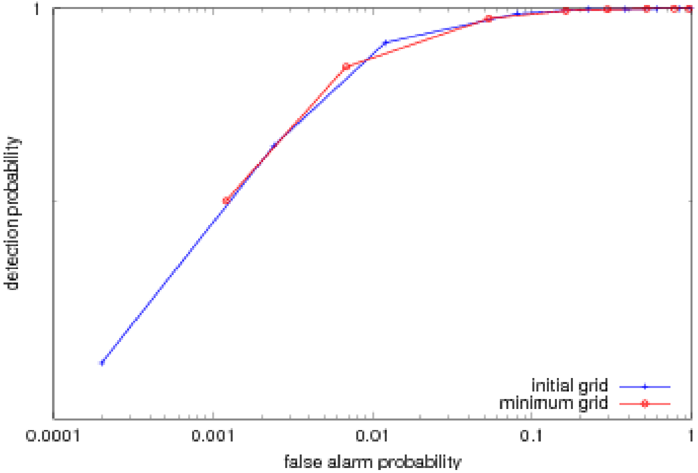
Extra Slides

Performance

Three detector network:

Signal: Monochromatic burst $h(t) = A(t)\exp(i\phi(t))$

Sampling freq: 4096 Hz Signal freq: 150 Hz



Grid is tight

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