# Double Optical Spring

## Tranquilizer in translational mode

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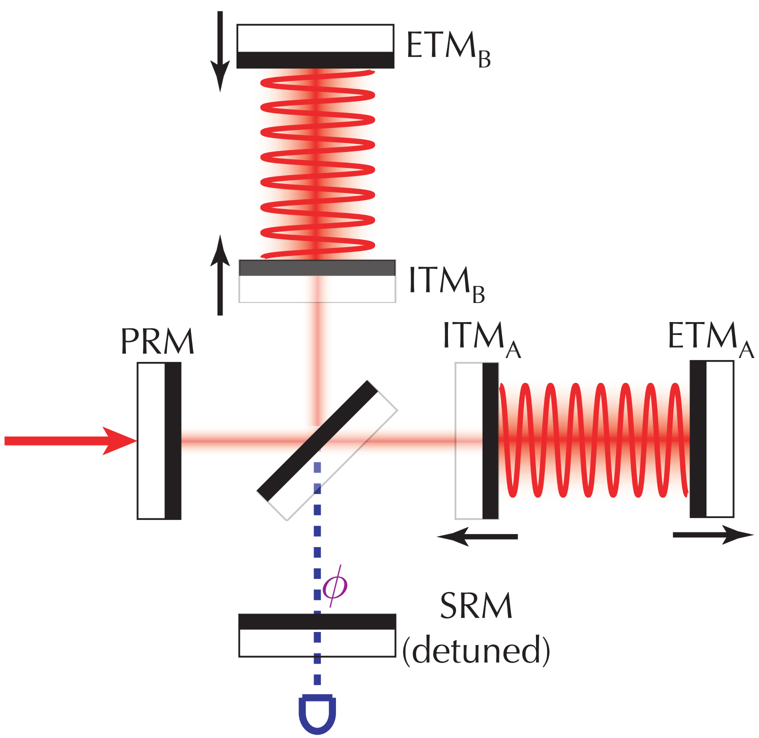
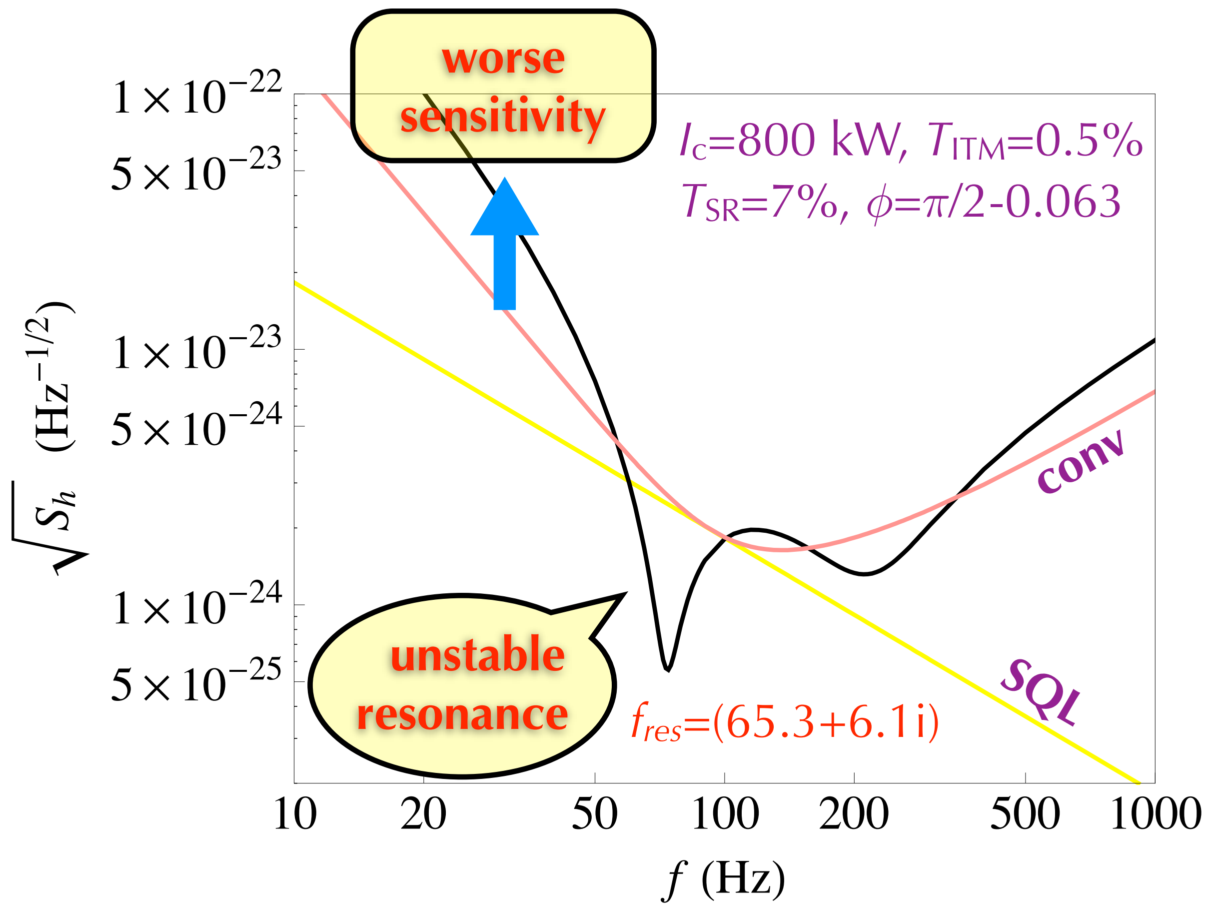
# Optical Spring: introduction and issues

- **Optical springs** in detuned signal recycling interferometers [Buonanno & Chen 00 & 01] differential mode can be viewed as single detuned cavity [Buonanno & Chen, 03].
- **Sensitivity:** *enhanced* around optical and mechanical resonances (beating SQL), but **suppressed** in other, especially low frequencies.
- **Instability:** optical spring resonance is unstable; in-band control does not impose fundamental noise [Buonanno & Chen, 00 & 01]




*optical spring connects ITM and ETM*



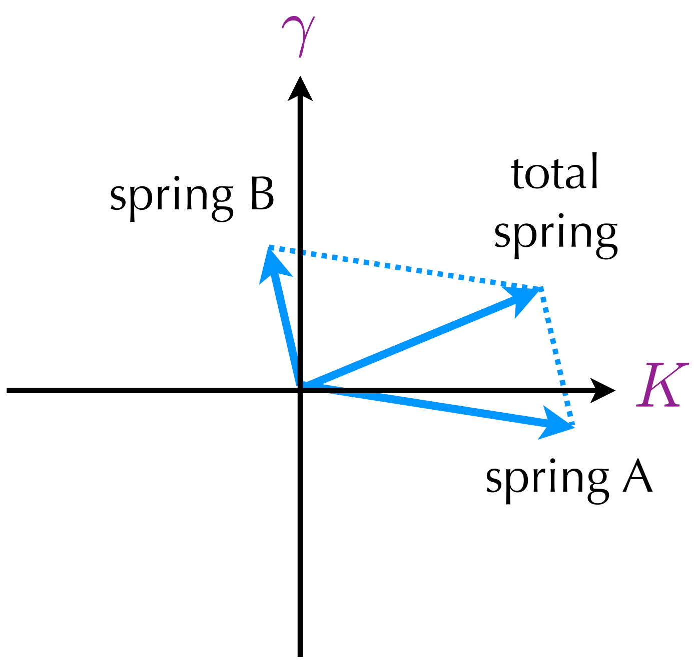
# Optical Stabilization without control

[Müller-Ebhardt, Rehbein et al., in preparation]

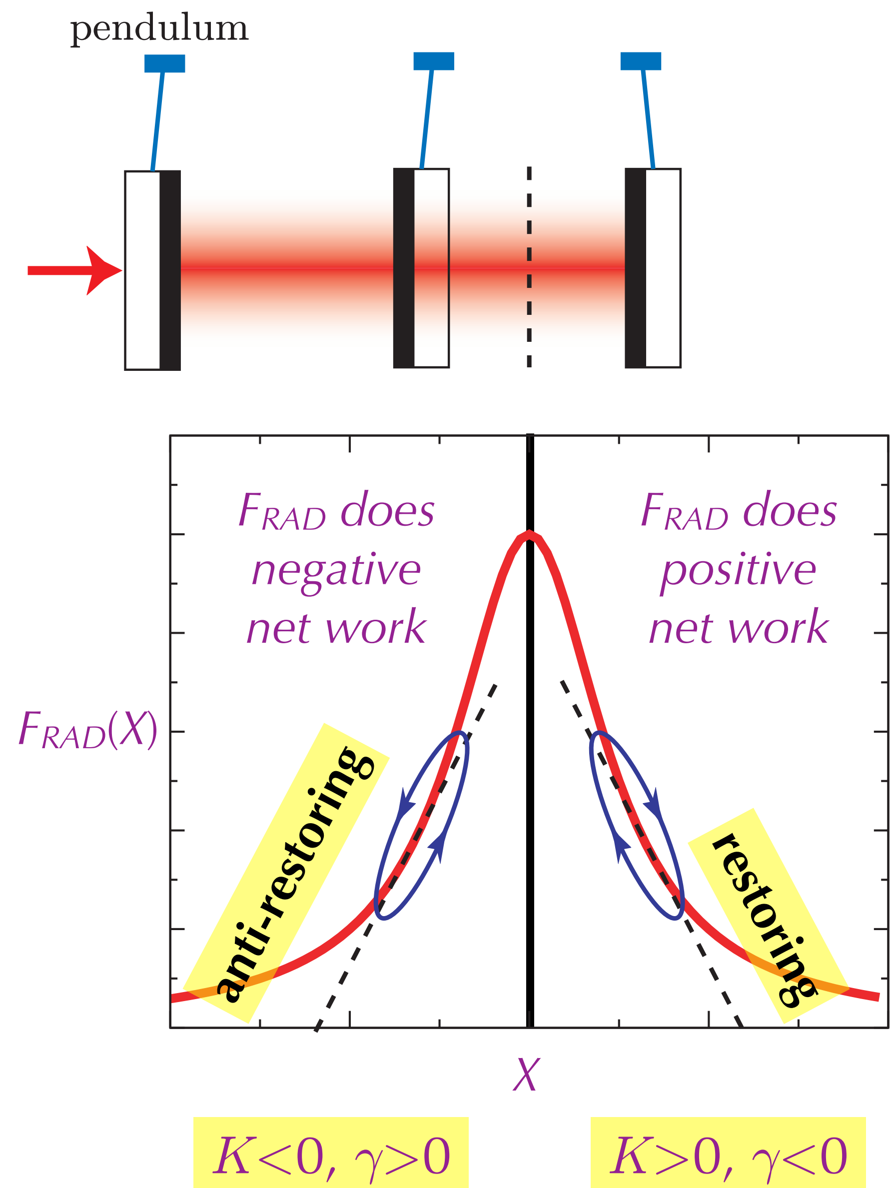
- Depending on sign of detuning, either
  - **restoring** + **anti-damping**
  - **anti-restoring** + **damping**
- In low frequencies, one can indeed write

$$F_{\mathrm{RAD}} = -KX - \gamma \dot{X} + \mathcal{O}(\Omega^2)$$

- Can we combine two springs, such that their total **good** features are positive?

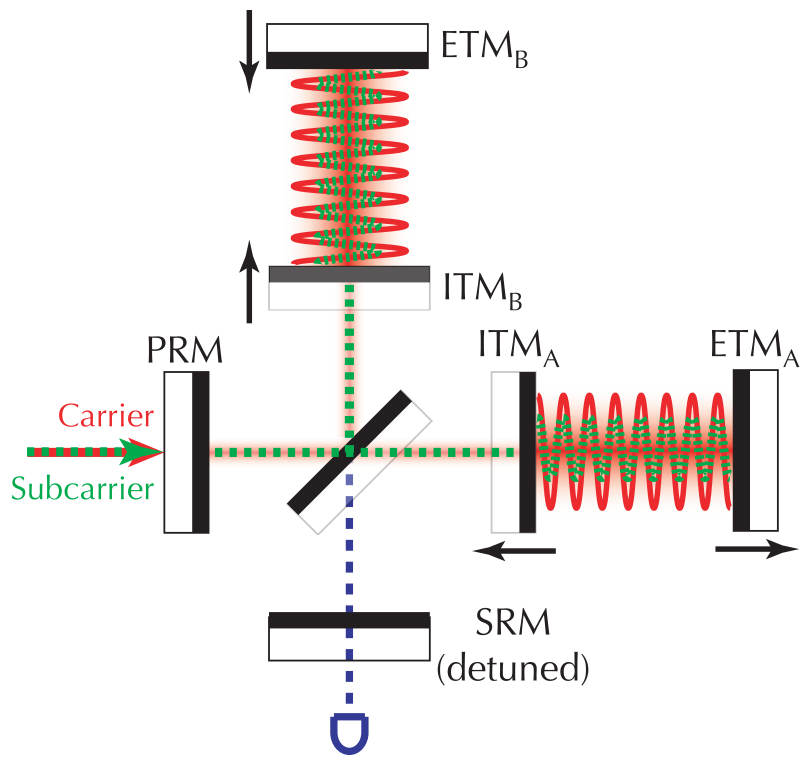


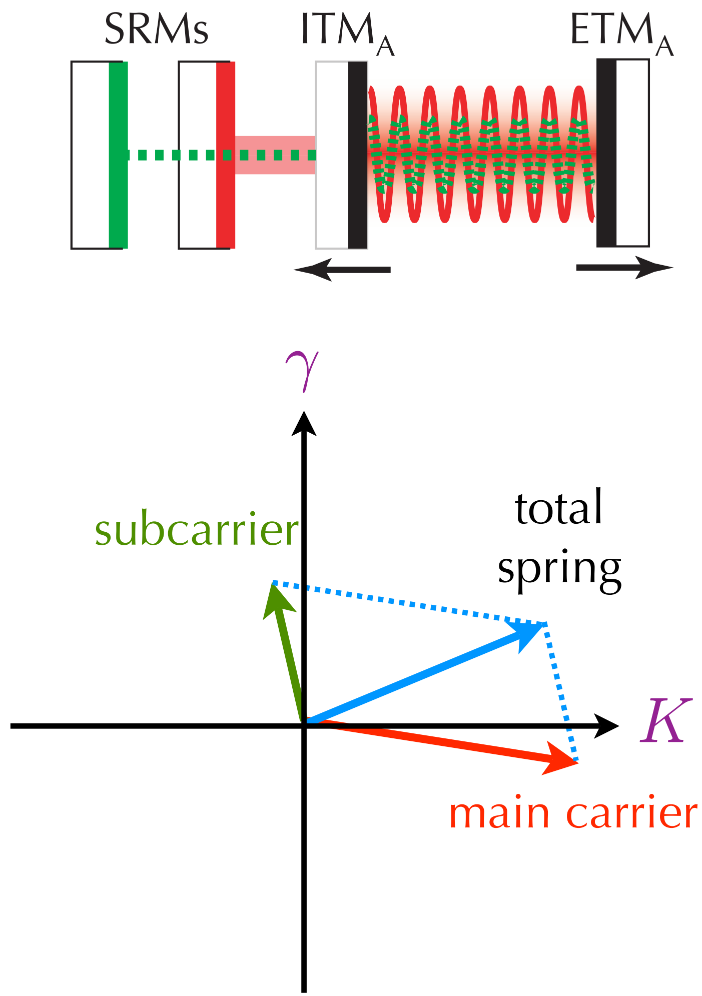

Similar to Braginsky & Vyatchanin's "tranquilizer" cavity used to stabilize elastic-mode parametric instability



# Double Optical-Spring Stabilization: Advanced LIGO

[Müller-Ebhardt, Rehbein et al., in preparation]



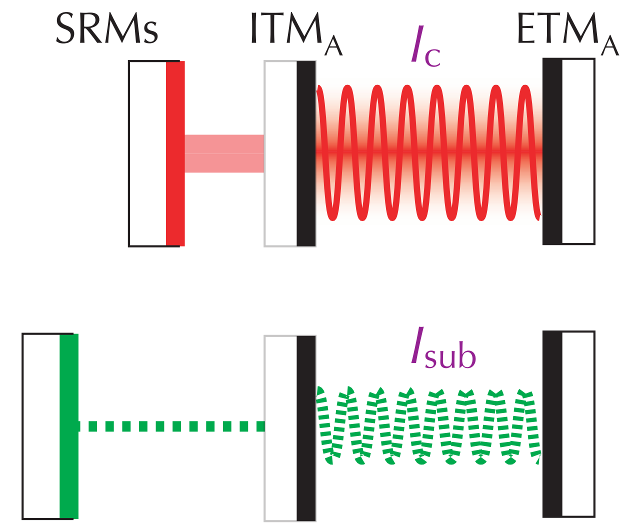



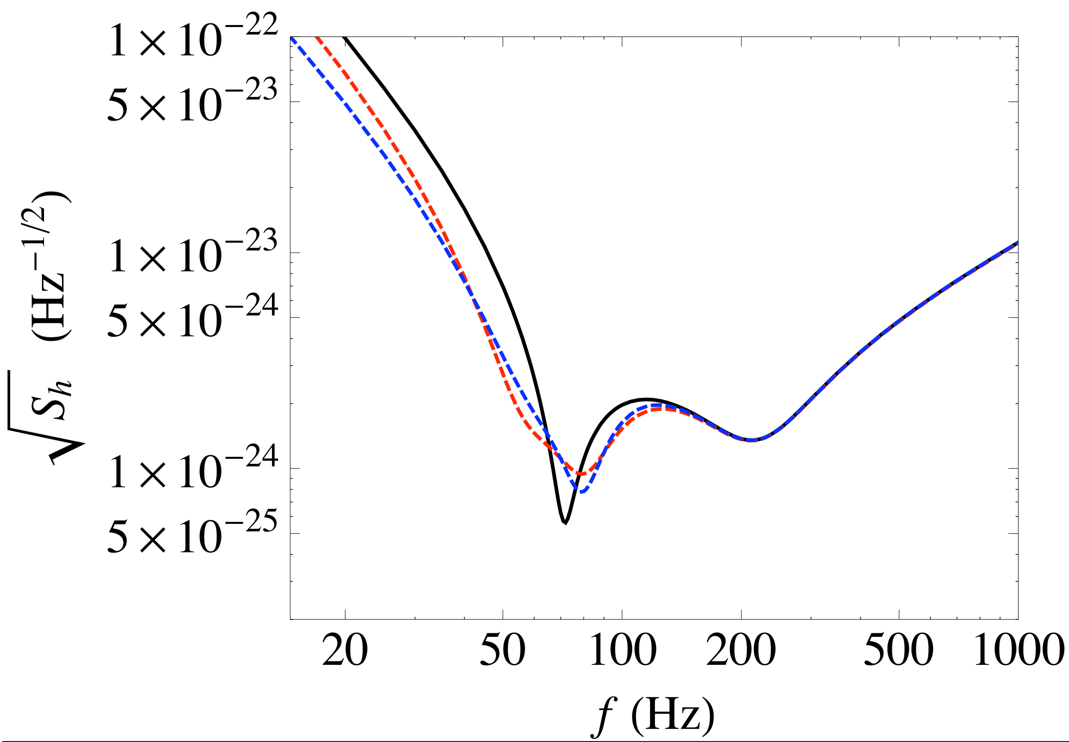

- Insert subcarrier that **resonates in the arms**, but have different **SR detuning phase** and **reflectivity** [perhaps different polarization ...]
- Carrier and subcarrier have different SR cavities, then each equivalent to a different single detuned cavity.

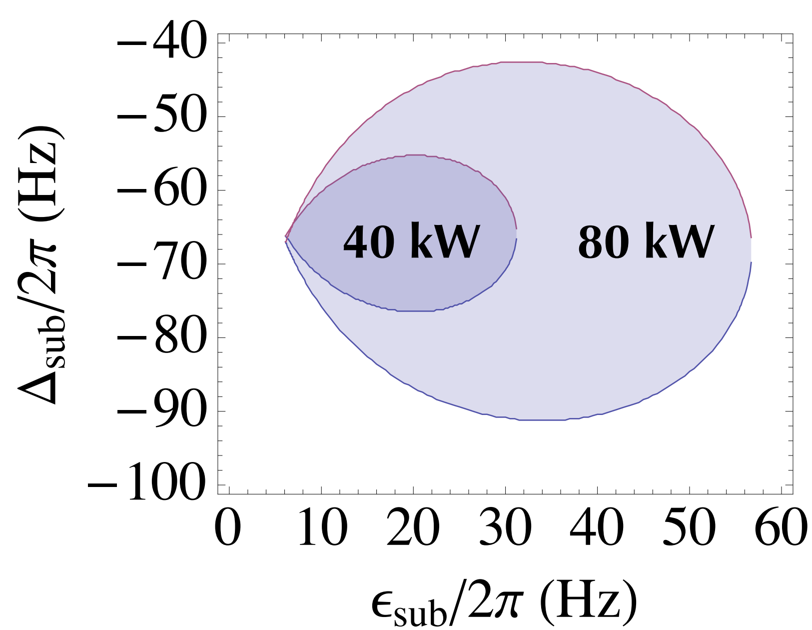
# Example Configuration

[Müller-Ebhardt, Rehbein et al., in preparation]

- Carrier: $I_c$=800 kW, $T_{ITM}$=0.5%, $T_{SR}$=7%, $\phi=\pi/2-0.063$
- Subcarrier: $I_{sub}$=40 kW, 80 kW, explore its **bandwidth** and **detuning freq,** ($\epsilon_{sub}$, $\Delta_{sub}$) and look for stable region
- We choose 40 kW, ($\epsilon_{sub}$, $\Delta_{sub}$)=$2\pi$(20,-66)
  - **stabilization:** (65.3+6.1i) → (58.9-6.6i)
  - **sensitivity**

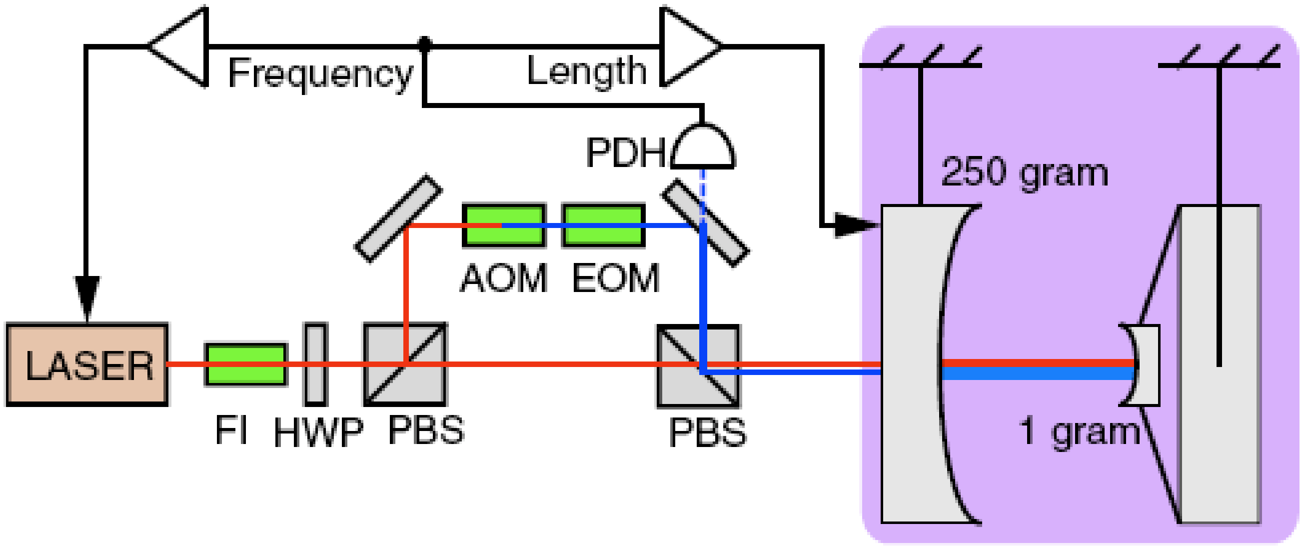


# Application to Ponderomotive Squeezer

## "An all-optical Trap for 1-gram Testmass"




Thomas Corbitt, Yanbei Chen, Edith Innerhofer, Helge Müller-Ebhardt, David Ottaway, Henning Rehbein, Daniel Sigg, Stanley Whitcomb, Christopher Wipf, and Nergis Mavalvala, PRL **98**, 150802 (2007)

# Optical-Spring Dynamics: Low Frequencies

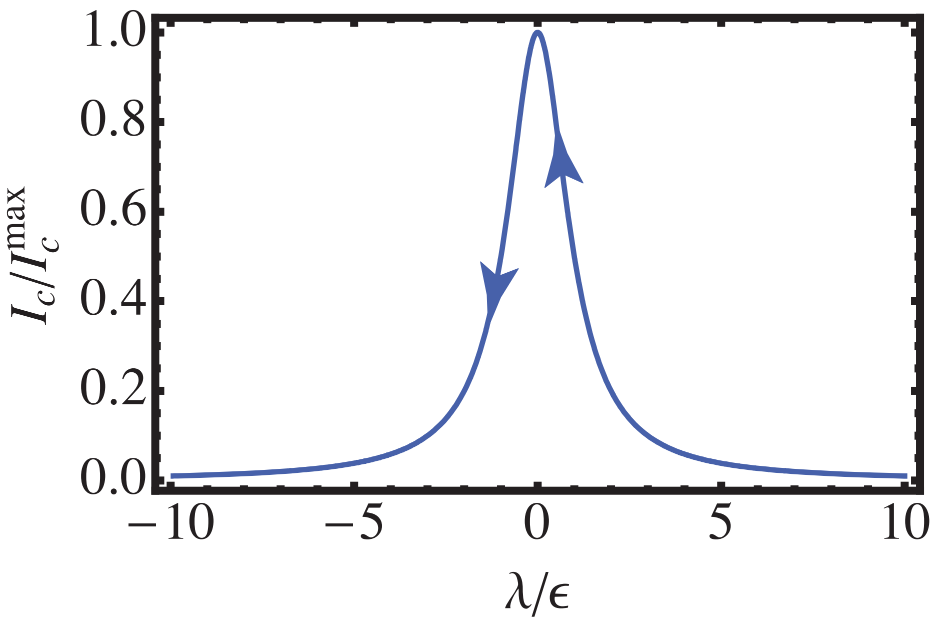
- If mechanical resonant frequency **much lower than** optical frequencies

$$K_{\mathrm{cplx}} \approx \frac{\mu\lambda\Theta^3}{\epsilon^2+\lambda^2}\left[1+\frac{2i\epsilon\Omega}{\lambda^2+\epsilon^2}\right] = K - i\Omega\gamma$$

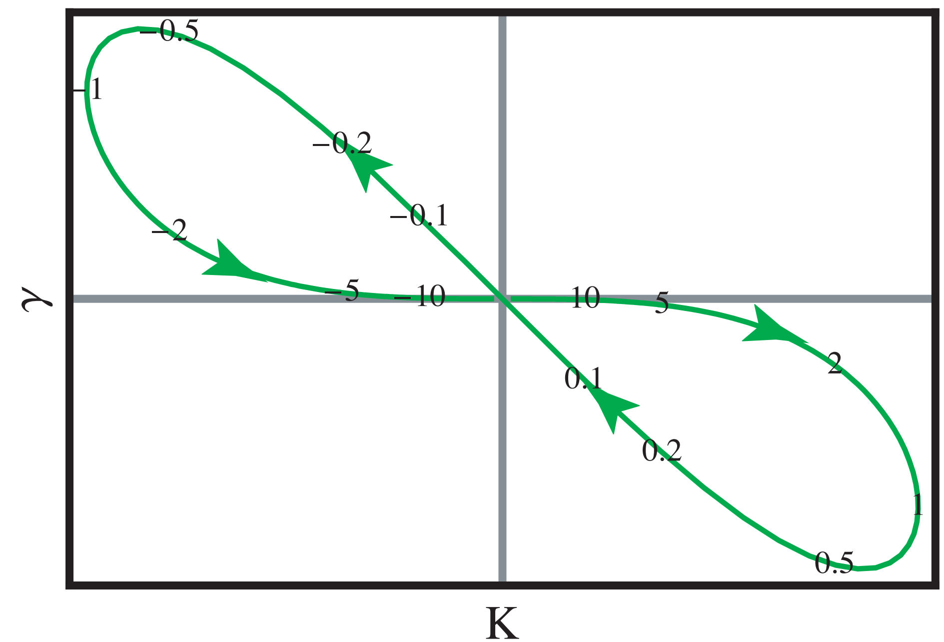
$K$, spring constant; $\mu$, reduced mass; $(\epsilon, \lambda)$, optical bandwidth and detuning freq

$$\Theta = \left(\frac{4\omega_0 I_c}{\mu L c}\right)^{1/3}, \text{ characteristic frequency}$$

$\omega_0$ laser freq; $I_c$, circulating power; $L$ cavity length; $c$, speed of light

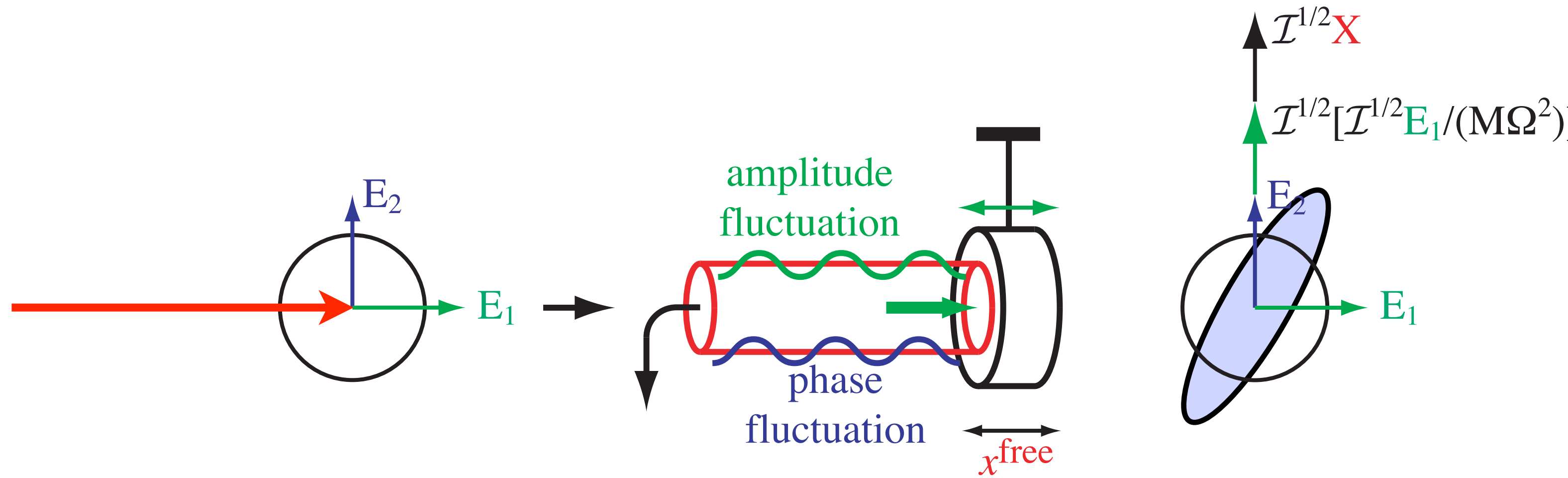



scan ratio between detuning and bandwidth keeping circulating power

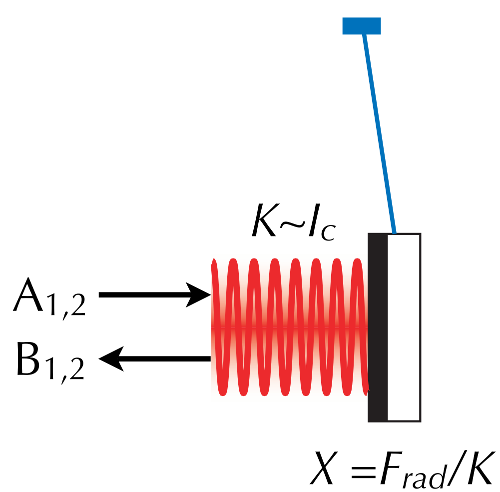


# Introduction to Ponderomotive Squeezing

- Ponderomotive Squeezing




- Advantage of using a spring [Corbitt et al., PRA **73**, 023801 (2006)]
  - Squeezing with constant factor and quadrature phase
  - Less susceptible to classical noises



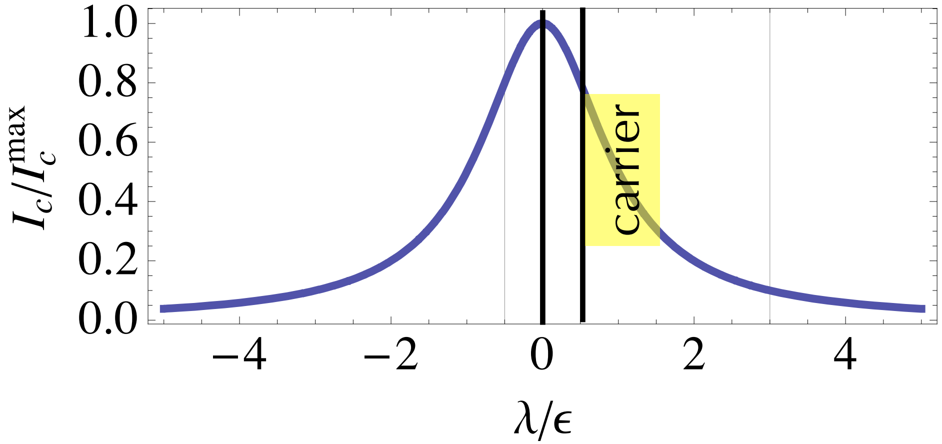

**Below mechanical resonance**

$$B_1 = A_1$$

$$B_2 = \frac{2\epsilon}{\lambda} A_1 + A_2 + \frac{2\Omega}{\Omega_{os}} \sqrt{\frac{\epsilon}{\lambda}} \frac{F}{F_{SQL}}$$

$$\Omega_{os} = \sqrt{\frac{\Theta^3(\lambda/\epsilon)}{\epsilon[1+(\lambda/\epsilon)^2]}} \qquad \Theta = \left(\frac{8\omega_0 I_c}{\mu L c}\right)^{1/3}$$

# Example Configuration

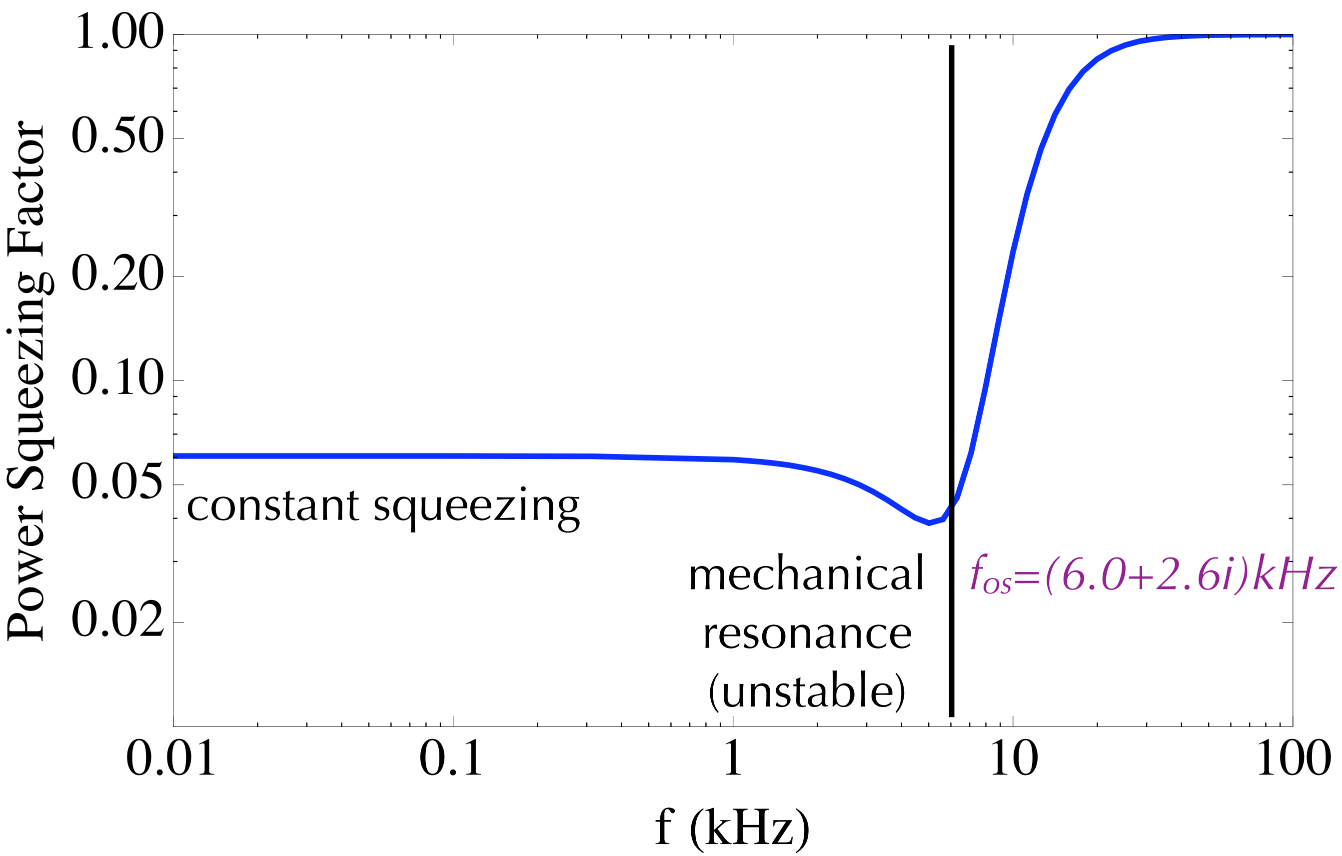



$M$ = 1 gram, $L$ = 1 meter, $I_0$ = 1.1 W, $T_I$ = 800 ppm

$(\epsilon, \lambda)/(2\pi)=(10,5)\,\text{kHz}.$

$I_c$ = 8.6 kW

$\Theta/(2\pi) = 12\,\text{kHz}$

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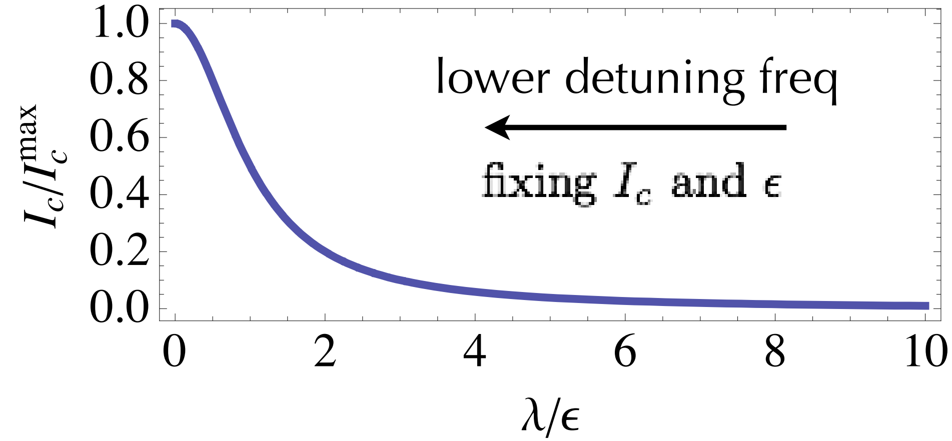
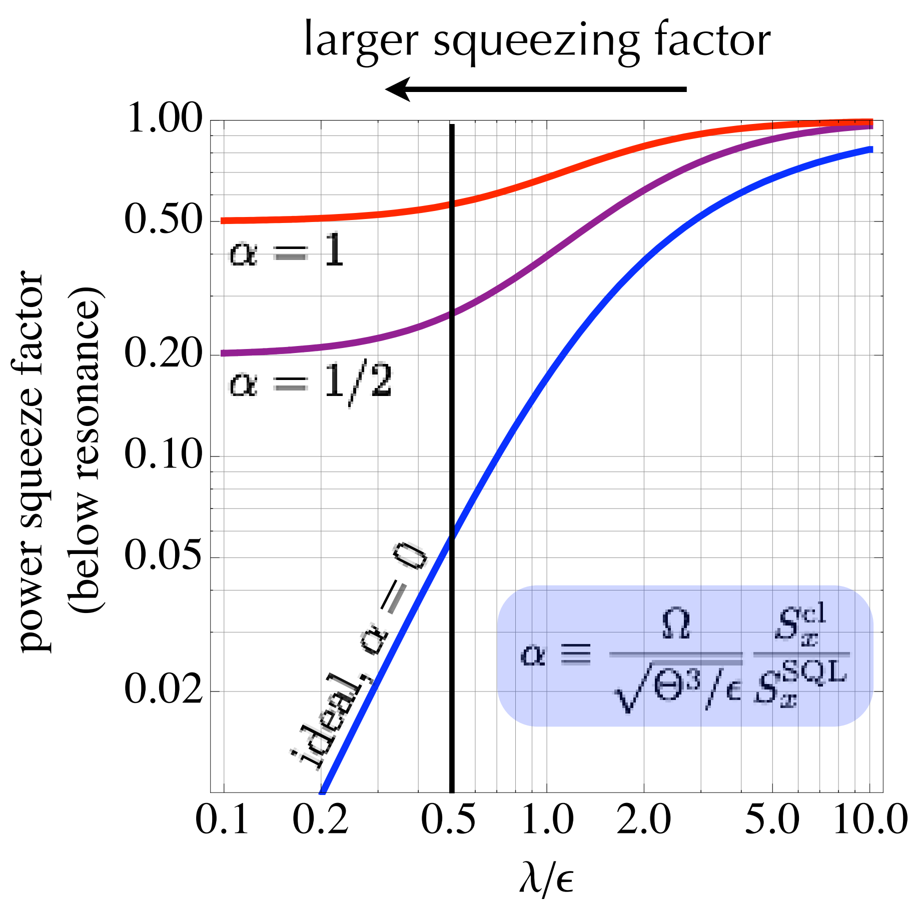
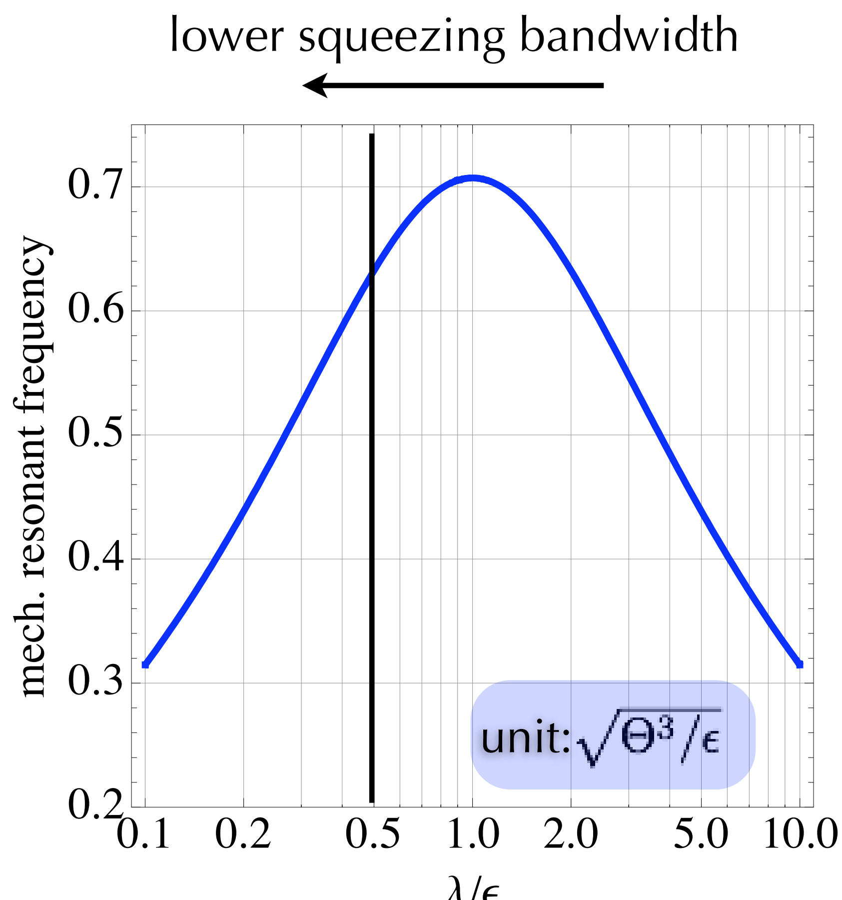
# Scaling Relations: Low-Frequency Approximation



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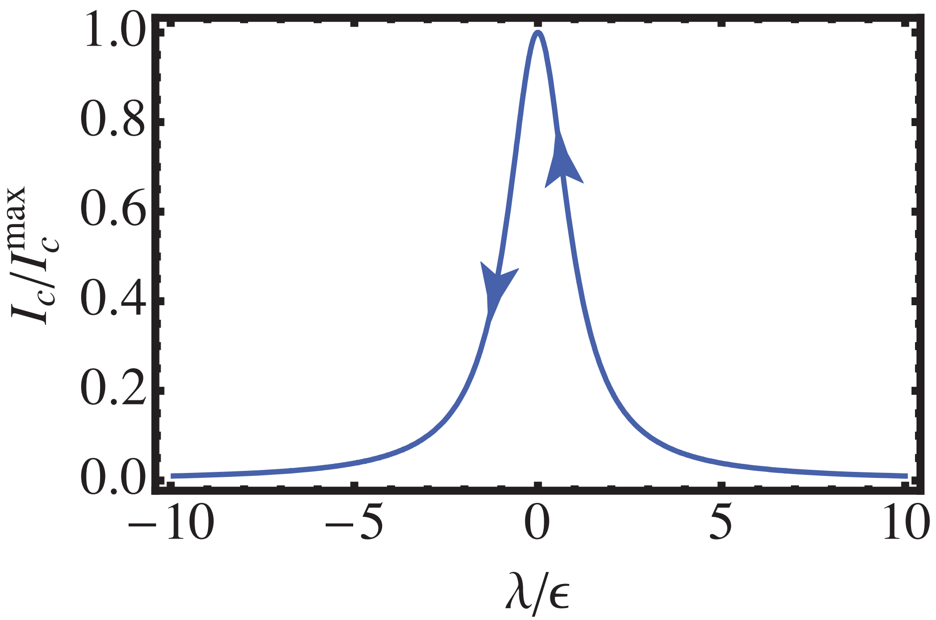


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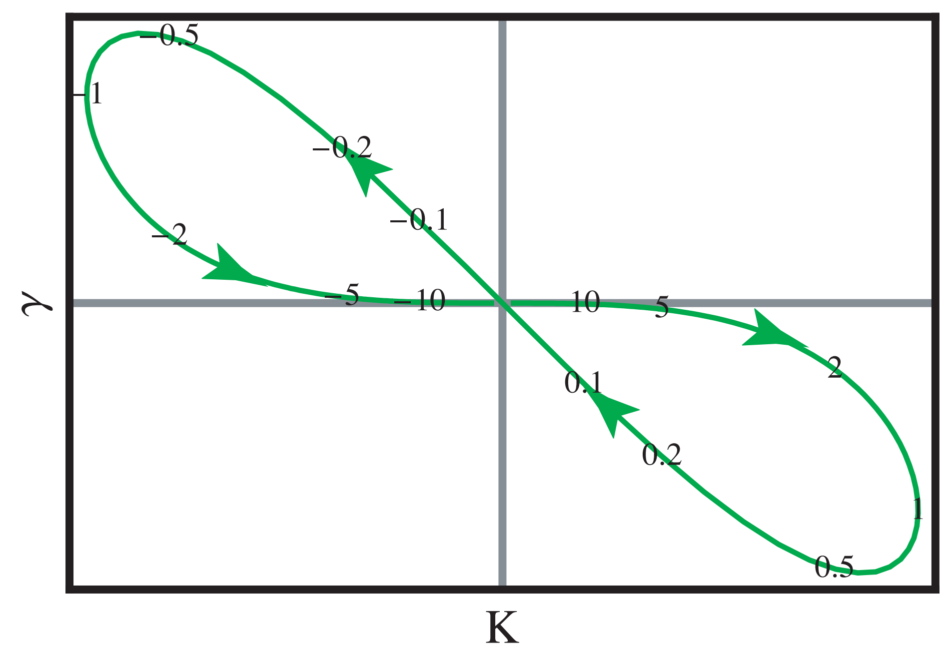
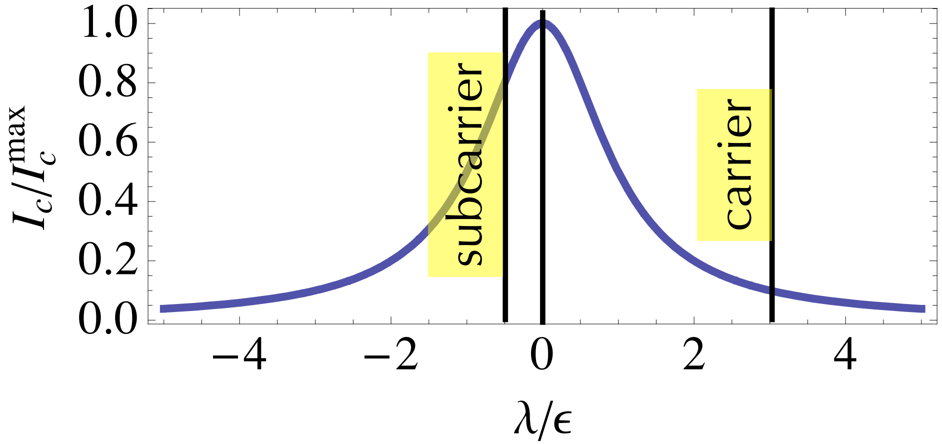
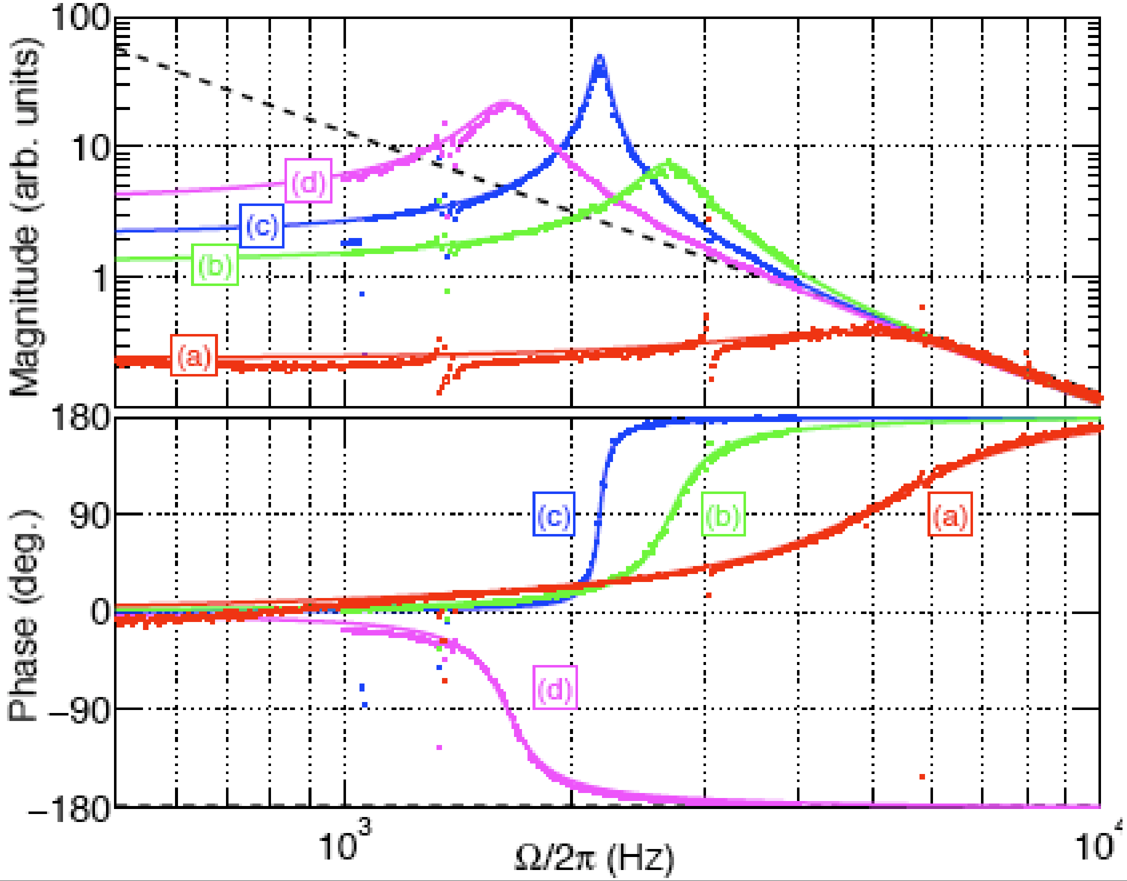


# Stabilization and Squeezing




$M$ = 1 gram, $L$ = 1 meter, $(I_1, I_2)$ = (5,0.5)W
$T_I$ = 800 ppm
$(\epsilon, \lambda_1, \lambda_2)/(2\pi) = (10, 30, 5)\,\mathrm{kHz}$

$(I_{c1}, I_{c1})$=(4.6,3.9)kW



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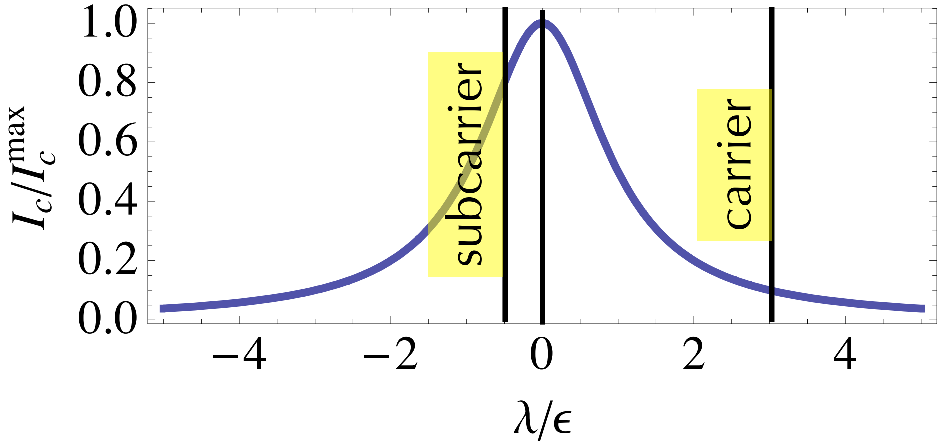


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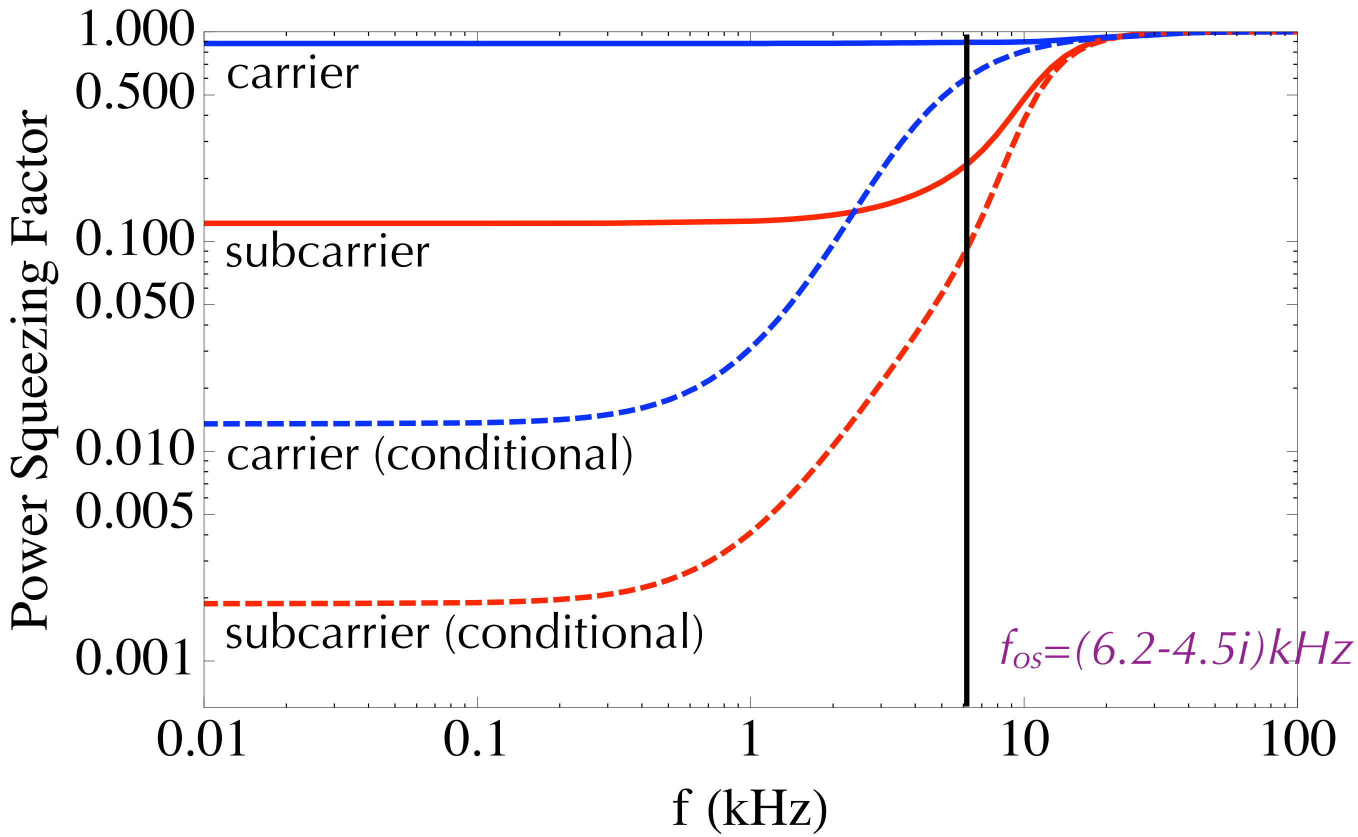


subcarrier is much more squeezed!!

conditioning allows much more squeezing!!

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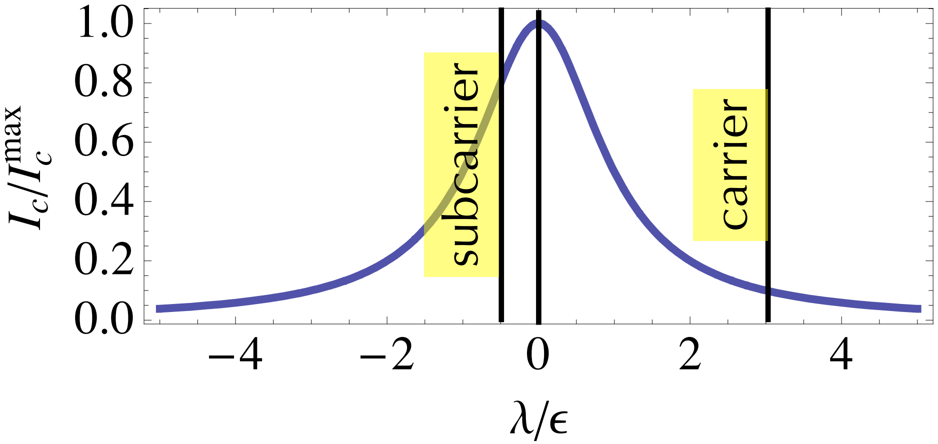


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squeezing entanglement

$$\begin{bmatrix} B_1^{(1)} \\ B_2^{(1)} \\ B_1^{(2)} \\ B_2^{(2)} \end{bmatrix} = \begin{bmatrix} 1 & & & \\ \kappa_{(1)}^2 & 1 & \kappa_{(1)}\kappa_{(2)} & \\ & & 1 & \\ \kappa_{(1)}\kappa_{(2)} & & \kappa_{(2)}^2 & 1 \end{bmatrix} \begin{bmatrix} A_1^{(1)} \\ A_2^{(1)} \\ A_1^{(2)} \\ A_2^{(2)} \end{bmatrix}$$

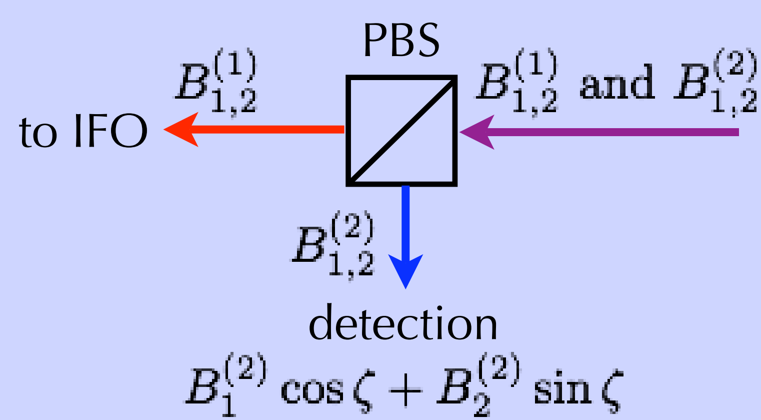
entanglement squeezing

$B_{1,2}^{(1)}$, mixed state
$B_{1,2}^{(2)}$: mixed state
$B_{1,2}^{(1)}, B_{1,2}^{(2)}$: pure state

"Conditioning"

Pure state of $B_{1,2}^{(1)}$ is recovered, if we measure

$$B_1^{(2)} \cos\zeta + B_2^{(2)} \sin\zeta$$



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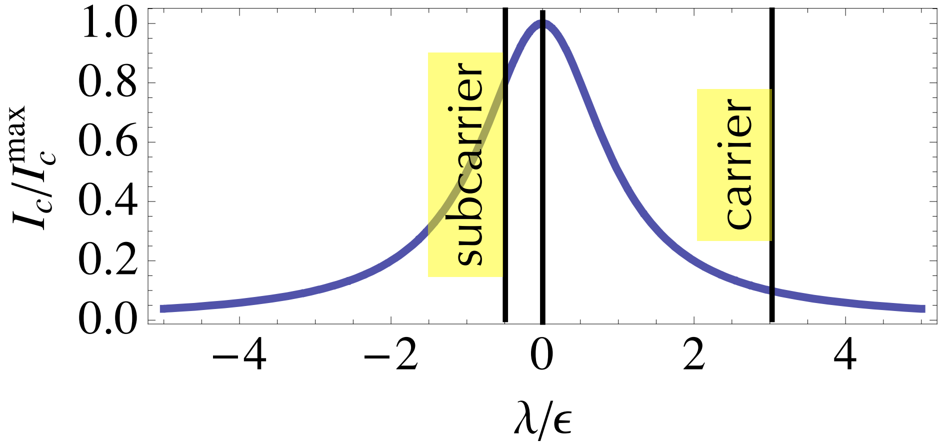


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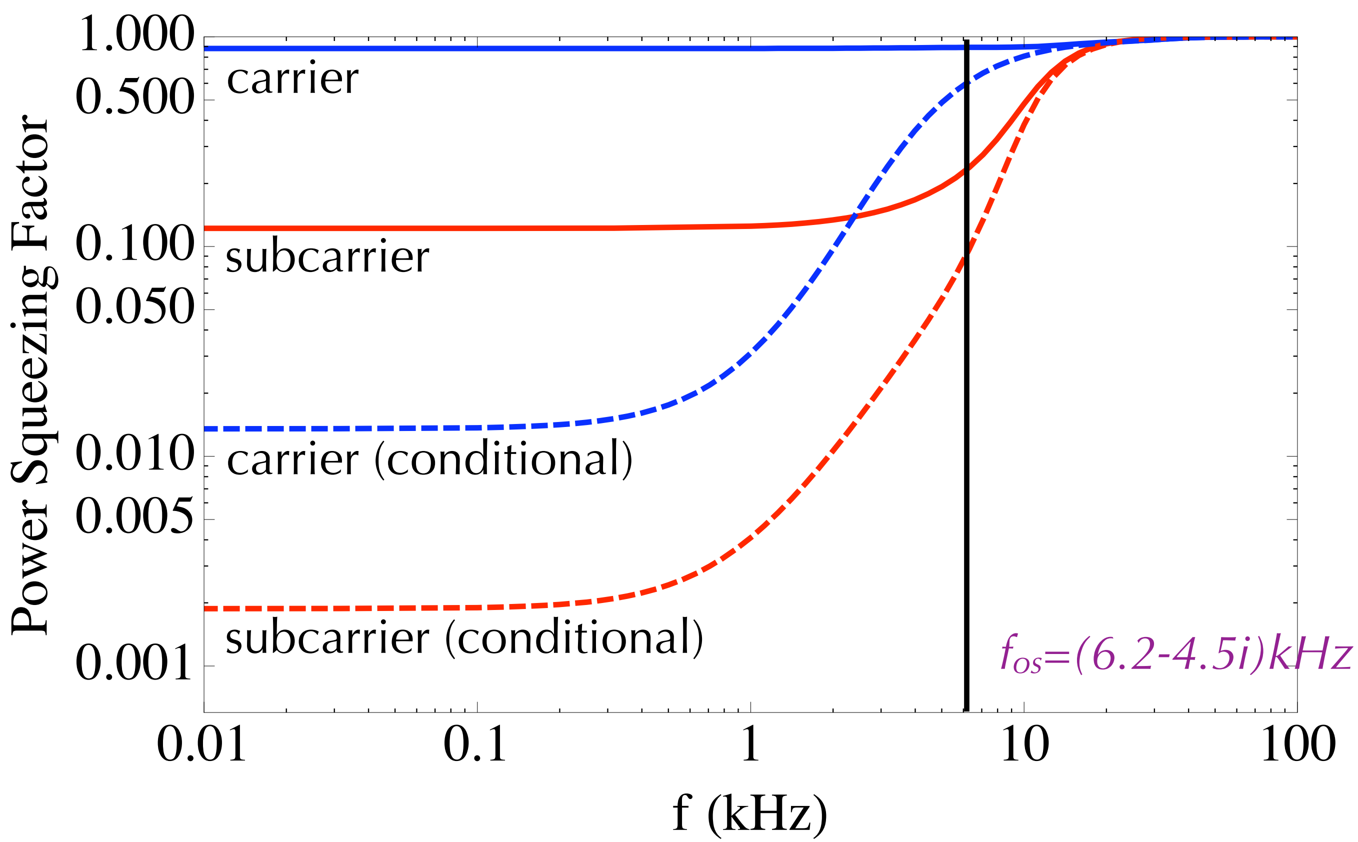


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