Precursors of Parametric Oscillatory Instability in Gravitational Detectors

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Perth, July 16, 2007

Outline

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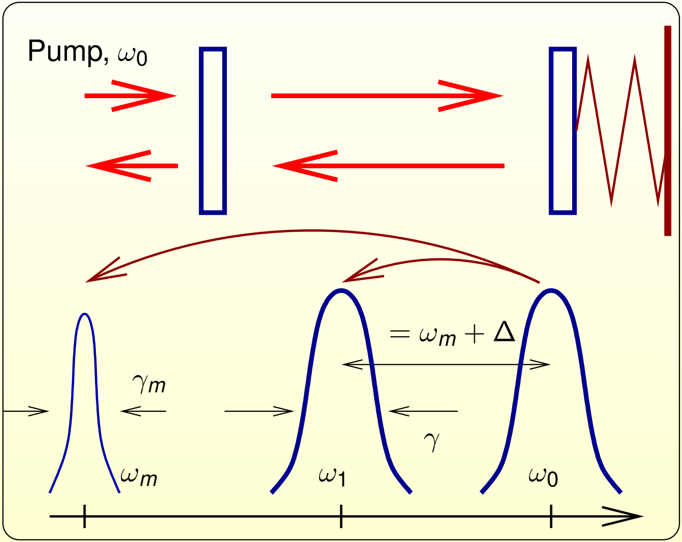
Effect of Parametric Oscillatory Instability in FP Cavity



Condition of Parametric Instability in FP Cavity:

Qualitative consideration:

- Detuning $\Delta = \omega_0 - \omega_1 - \omega_m$ is small: $\Delta \ll \gamma$.
- Power W_0 circulating inside cavity is equal to the threshold value W_c: $W_0 = W_c$[a].
- We have important condition: $\gamma_m \ll \gamma$. Flow of energy partially compensates dissipated power in elastic mode, i.e. efffective relaxation rate $\gamma_m^{\text{eff}} \to 0$ as $W_0 \to W_c$.
- In approximation of given amplitude of main mode (i.e. unlimited pump): elastic oscillations amplitude and optical power in Stokes mode rise exponentially.

[a] V. B. Braginsky, S. E. Strigin, and S. P. Vyatchanin, *Physics Letters* **A287**, 331 (2001); gr-qc/0107079.

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The aim of consideration

Precursors

The analysis of PI dynamics in FP cavity as pump power increases close to threshold. The knowledge of such dynamics will allow experimenter to look for *precursors* of PI as alarm signals.

Variant of tranquilizer

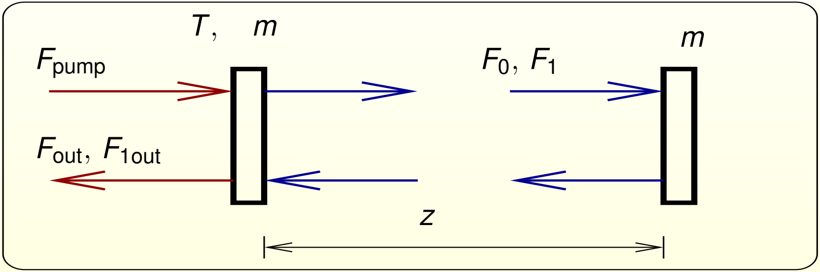
One can prevent PI evolution, for example, by introducing the low noise damping into risky (candidate for PI) elastic mode.

Precursors observation and tranqualizer in situ

It is important to test methods to prevent PI in full scale detectors, for example, in GEO or Gingin.

Outline

We analyze Parametric instability in FP cavity:



Model

We take into account evolution of main mode fixing pump power, but not power circulating inside.
We consider the mirrors as elastically identical, i.e. the elastic modes in both mirrors have the same frequencies and spatial distribution.

Model

Equations to analyze:

$$\dot{F}_0^* + \gamma_0 F_0^* = \gamma_0 B_0 - \gamma_1 r F_1^* \, Z^* \, e^{-i\Delta t}, \qquad (1)$$

$$\dot{F}_1 + \gamma_1 F_1 = \gamma_1 F_0 Z^* \, e^{-i\Delta t}, \qquad (2)$$

$$\dot{Z}^* + \gamma_m Z^* = \gamma_m V \, F_0^* F_1 \, e^{i\Delta t}, \qquad (3)$$

$$\Delta = \omega_0 - \omega_1 - \omega_m \quad V = \frac{2\Lambda\omega_1}{cL\gamma_1 \, m\omega_m\gamma_m}. \qquad (4)$$

Overlapping factor:

$$\Lambda = \frac{V_m \left| \int \mathcal{A}_{0in} \, \mathcal{A}_{1in}^* \, u_\perp \, d\vec{r}_\perp \right|^2}{\int |\mathcal{A}_{0in}(\vec{r}_\perp)|^2 \, d\vec{r}_\perp \; \int |\mathcal{A}_{1in}(\vec{r}_\perp)|^2 \, d\vec{r}_\perp \; \int |\vec{u}(\vec{r})|^2 \, d\vec{r}}.$$

Important condition

$$\gamma_m \ll \gamma_0,\ \gamma_1$$

Estimates of optical and elastic modes:

$$\gamma_0,\ \gamma_1 \geq 5\ \text{s}^{-1},\ \gamma_m \geq 10^{-2}\ \text{s}^{-1}. \tag{5}$$

It means that optical fields react on slow variations of elastic mode amplitude practically in a moment.

Recall the condition of parametric instability

PI condition

Power $W_0 \equiv |F_0|^2$ of main mode circulating inside cavity have to be equal to thresold value W_c, i.e. $W_0 = W_c$:

$$W_c V_\Delta = 1, \quad V_\Delta = \frac{2\,\Lambda\,\omega_1}{cL\,\gamma_1\,m\,\omega_m\gamma_m\left(1+\Delta^2/\gamma_1^2\right)}. \tag{6}$$

What will be close and above threshold?

What will be with behavior (dynamic) of our system when pump power increases to close and above theshold?

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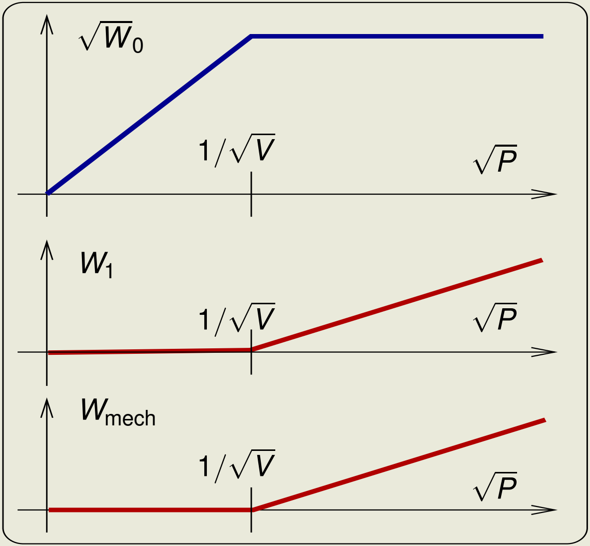
Steady state solutions

The more realistic approximation of given pump P.

One can consider the parametric instabiility as phase transition of system consisting of three oscillators non-lineary coupled with each other.



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Linearization for small amplitudes

In approximation $\gamma_0,\ \gamma_1 \gg \gamma_m$

one can obtain that effective relaxation rate of elastic mode go to zero at pump becames close to threshold (i.e. $VP \to 1$):

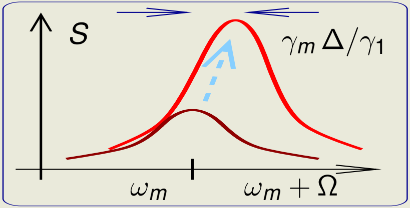
$$\gamma_m^{\text{eff}} = \gamma_m \big(1 - VP\big). \tag{7}$$

Precursors of PI

Far from PI threshold fluctuational forces in elastic mode produce peak of spectral density in output port. Close to threshold this peak increases:

$$S_{\text{far from threshold}}(\Omega) \sim \frac{S_{\text{fluct.}}(\Omega)}{\gamma_m^2 + \Omega^2},$$

$$S_{\text{close to threshold}}(\Omega) \sim \frac{S_{\text{fluct.}}(\Omega)}{(\gamma_m^{\text{eff}})^2 + \Omega^2}$$



Nonlinearity

Infinity ?

Peak of $S(\Omega)$ increases infinitely at threshold as $\gamma_m^{\text{eff}} \to 0$. This infinity originates exclusively in mathematical model, in particular, from linearization of initial set of equations. For correct consideration we should return to initial set without linearization.

Simplest case of zero detuning ($\Delta = 0$)

Taking into account condition $\gamma_m \ll \gamma_0,\ \gamma_1$ we may assume that optical fields react on slow variations of elastic mode amplitude practically in a moment — it means that we can omit time derivatives in equations for optical fields.

Nonlinearity (cont.)

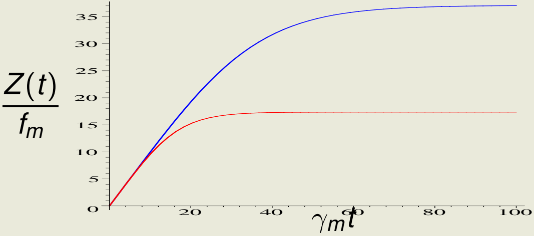
Equation for elastic mode amplitude Z. Zero detuning.

Pump close to threshold (i.e. $VP \to 1$). Resonance force with amplitude f_m:

$$\dot{Z} + \gamma_m Z \left(1 - \frac{VP}{\left(1 + \frac{r\gamma_1}{\gamma_0} Z^2\right)^2}\right) = \gamma_m f_m\,, \quad Z(t \to \infty) \simeq \sqrt[3]{\frac{f_m \gamma_0}{\gamma_1}}$$

Plots with threshold pump: $VP = 1$

Numerically calculated plots $Z(t)/f_m$ for: $r\gamma_1 f_m^2/\gamma_0 = 10^{-3}$ (lower plot) and $r\gamma_1 f_m^2/\gamma_0 = 10^{-5}$ (upper plot).



Discussion

Proposed "cures" to avoid the parametric instability:

(i) to change the mirror shape[a];
(ii) to introduce low noise damping[b];
(iii) to heat the test masses in order to vary curvature radii of mirrors and hence to control detuning and decrease overlapping factor[c].

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[a] V. B. Braginsky, S. E. Strigin and S. P. Vyatchanin, *Physics Letters* **A305**, 111 (2002).

[b] V. B. Braginsky and S. P. Vyatchanin, *Physics Letters* **A293**, 228 (2002). S.W. Schediwy, C. Zhao, L. Ju, D.G. Blair, P. Willems, submitted to Phys. Rev. Lett.

[c] C. Zhao, L. Ju, J. Degallaix, S. Gras, and D. G. Blair, Phys. Rev. Lett. **94**, 121102 (2005).

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Dynamic of precursors

Knowledge of detail precursor's dynamic allows to avoid PI more effectively. In particular, using dependence of elastic peak on circulating power one can separate PI precursors from elastic peaks increased by other reason.
In addition, one can vary detuning of anti-symmetric mode (by positioning signal recycling mirror) to observe precursors of PI *in situ*.[a]

[a] A.G. Gurkovsky, S.E. Strigin and S. P. Vyatchanin, Physics Letters A **362**, 91 (2007); arXive: gr-qc/0608007.

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Damping of elastic mode

Tranquilizer[a] introduces low noise damping in elastic mode of mirror. (Mirror is one mirror of additional FP cavity, pumped by additional laser detuned from optical resonance — it produces negative optical rigidity into elastic mode motion combined with low noise damping.) Unfortunately, the direct use of such tranquilizer requires too high pump in additional FP cavity. D. Blair with colleagues propose to enhance damping through increasing time delay in servo loop, controlling additional FP cavity[b].

[a] V. B. Braginsky and S. P. Vyatchanin, *Physics Letters* **A293**, 228 (2002).

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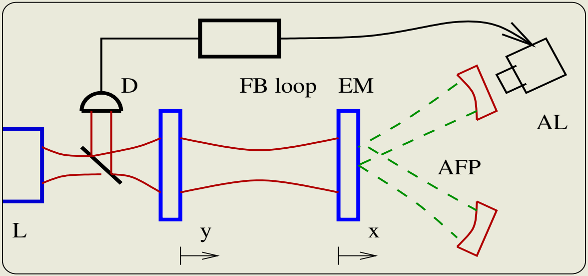
Low noise damping by feed back

Possible scheme



Main FP cavity (FP) is pumped by laser (L). Information on elastic mode oscillations in end mirror (EM) from detector D is used to modulate intensity of additional laser (AL) through feedback (FB loop). When this modulation is proportional to velocity then radiation pressure force will be also proportional to velocity, i.e. we have damping. The additional FP cavity (AFP) is tuned in resonance with additional laser.

Low noise damping by feed back (cont.)

Information on oscillations of mirror surface:

a) through detector D with **high accuracy but delayed** by approximately relaxation time of main FP cavity which in Advanced LIGO — about 0.1 ... 0.5 sec;
b) through the wave reflected from additional FP cavity with **low accuracy but practically instant**.

The diameter of light beam in additional FP cavity should be about 1 mm.

The possibility to apply light pressure force to different places of surface and thereby tranquilizing elastic modes with different distributions of displacement over the mirror surface.

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Low noise damping by feed back (cont.)

Additional noise through two mechanisms:

a) the electronic noise of the feed back;
b) the amplitude fluctuations of additional laser (producing fluctuations of light pressure).
Feed back noise may be excluded — one can filter signal in feed back loop only in the range of elastic mode frequencies (30 – 120 kHz) and do not transfer work frequency range of the gravitational signal (50 – 500 Hz).

Fluctuations of radiation pressure noise

will be small enough.

$$F_{fl} = \frac{2}{c}\sqrt{\frac{W\,\hbar\omega_0}{\tau}} \geq \frac{m\omega_m^2}{Q_m}\sqrt{\frac{\kappa_B T}{m\omega_m^2}}$$

Assuming $m = 40$ kg, $\omega_m = 10^6\ \mathrm{s}^{-1}$, $Q_m \simeq 10^4$, $T = 300$ K we estimate $W \simeq 10$ Watts. To be on the safe side we assume $\mathbf{W \simeq 10^3}$ Watt. The radiation pressure force fluctuations F_{fl} produced by photon shot noise on frequency $\omega_{\mathrm{grav}} \simeq 10^3\ \mathrm{s}^{-1}$:

$$\Rightarrow\ h_{\mathrm{fl.grav}} \simeq \frac{x_{\mathrm{fl.grav}}}{L} \simeq \frac{F_{fl}}{m\omega_{\mathrm{grav}}^2 L} \simeq 10^{-28},$$

$$h_{\mathrm{fl.grav}} \ll h_{\mathrm{Adv.LOGO}} \simeq 10^{-22}$$

GEO and AIGO as testing area?

To invent methods of tranqualizing PI:

a) invention and test of tranqualizing methods on laboratory prototypes;
b) To test *in situ* — GEO or AIGO are good candidates.

We have to enchance PI in GEO-600 or AIGO

Effective detuning of anti-symmetric optical mode

by displacement of SR mirror. Range of detuning is about free spectral range: for GEO $\Delta f_{frs} \simeq 600$ Hz — it is large enouph to choose suitable elastic mode with not small overlapping factor Λ_1.

Decrease of relaxation rate γ_- of anti-symmetric mode.

To increase the optical power circulating in arms or to decrease relaxation rate γ_- of anti-symmetric mode. Doubled mirror[a]: to place additional mirror parallel to SR mirror (they assemble short FP cavity).

$$T_{eff} = T_{sr} T_{add}/4 \text{ (anti-reson.) } \ldots 4 T_{sr} T_{add}/(T_{sr} + T_{add})^2 \text{ (reson.)}$$

[a] A.G. Gurkovsky and S.P. Vyatchanin, Physics Letters A, accepted for publication

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We need detail information on elastic modes

Numerical calculations of elastic modes

To attribute elastic modes from experiment readout we need detail information on frequencies and distributions of elastic modes. We need the accuracy at least:

$$\frac{\Delta\omega_m}{\omega_m} \leq \frac{\gamma_1}{\omega_m} \simeq 10^{-7} \div 10^{-5} \text{ (!)}.$$

How to improve accuracy of numerical calculations?

Preliminary results: COMSOL provides accuracy of elastic frequency calculations about 3×10^{-5} only for axial symmetric elastic modes (2D module). For non- axial symmetric mode (3D) the accracy is unsufficient.

Summary

Chance to avoid PI

The information about precursors combined with effective feed back tranquilizer will allow to avoid undesirable PI in Advanced LIGO (the danger forseen is half avoided).

Additional noise?

We may expect the appearance of new additional noises correlated with the effect of PI. These noises were not predicted by the analyzed model.

Outlook

Numerical calculation of elastic modes frequencies
with relative error $< 10^{-5}$

Acknowledgments

Many thanks to

Ju Li,
David Blair

for kind invitation to this wokshop,

Vladimir Braginsky,
Bill Kells,
Adrian Melissinos,
David Ottaway

for fruitful discussions.