



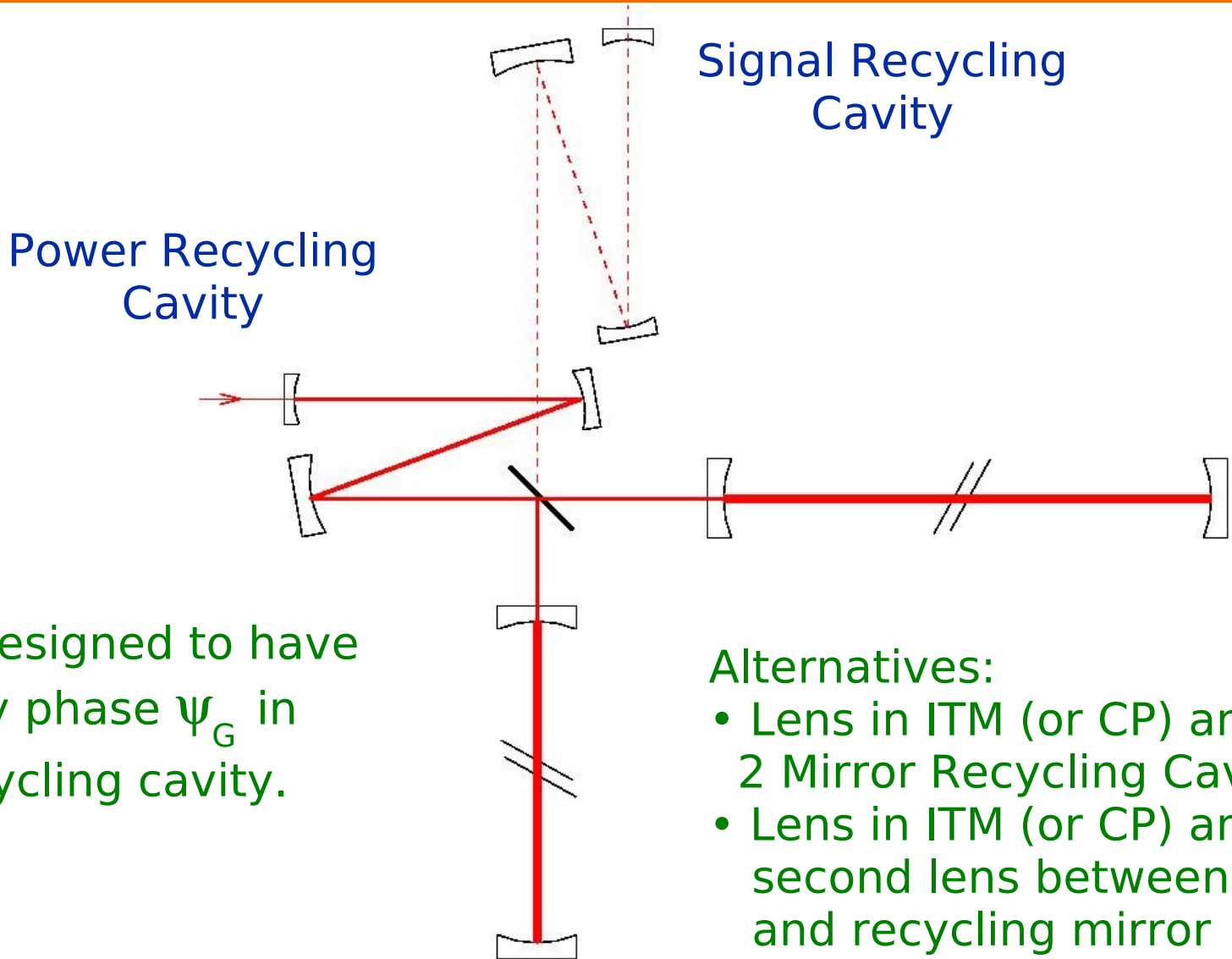
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**Parametric Instabilities  
and the geometry of the  
recycling cavities**

**LIGO-G070441-00-R**

**Guido Mueller**

**(Credit goes to Bill Kells,  
David Ottaway, Valera Frolov)**



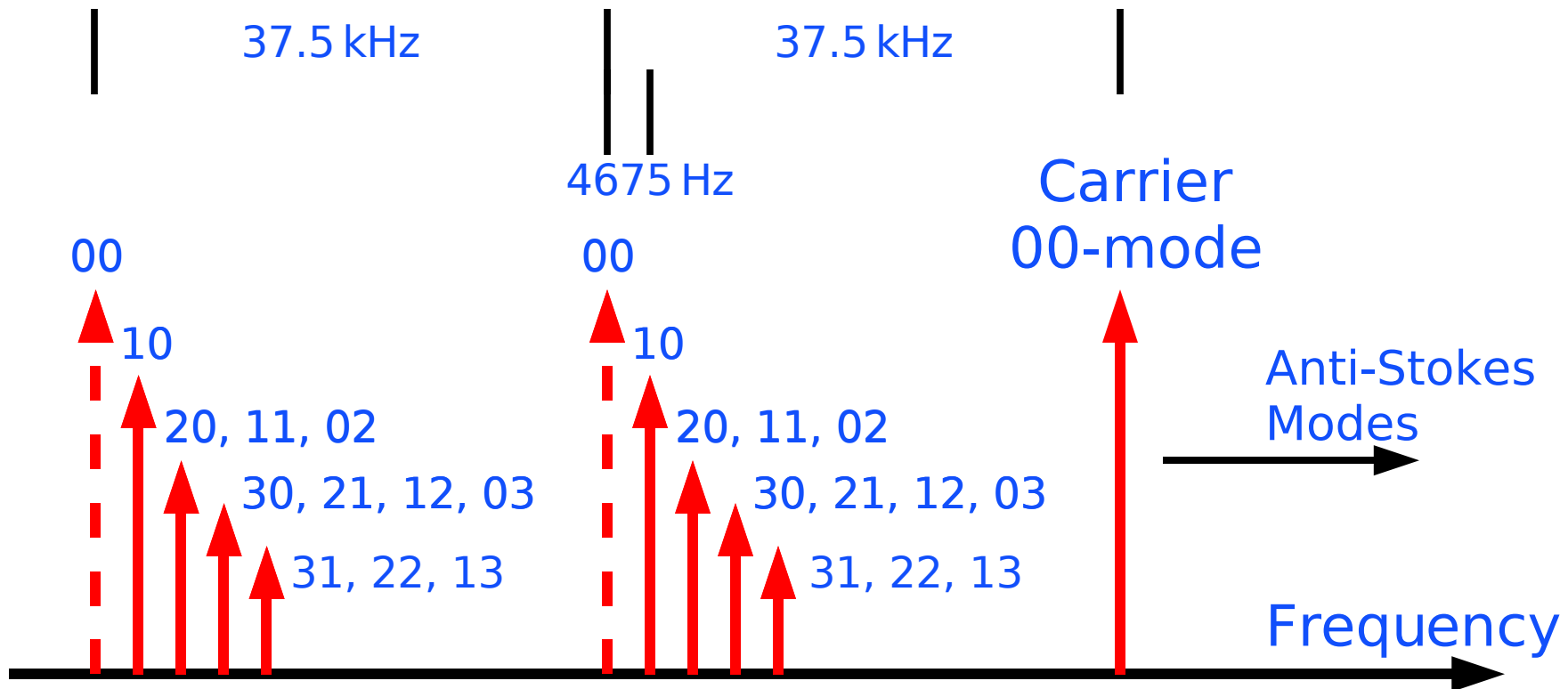
Can be designed to have any Gouy phase  $\psi_G$  in each recycling cavity.

Alternatives:

- Lens in ITM (or CP) and 2 Mirror Recycling Cavity
- Lens in ITM (or CP) and second lens between BS and recycling mirror



## PI-Spectrum:

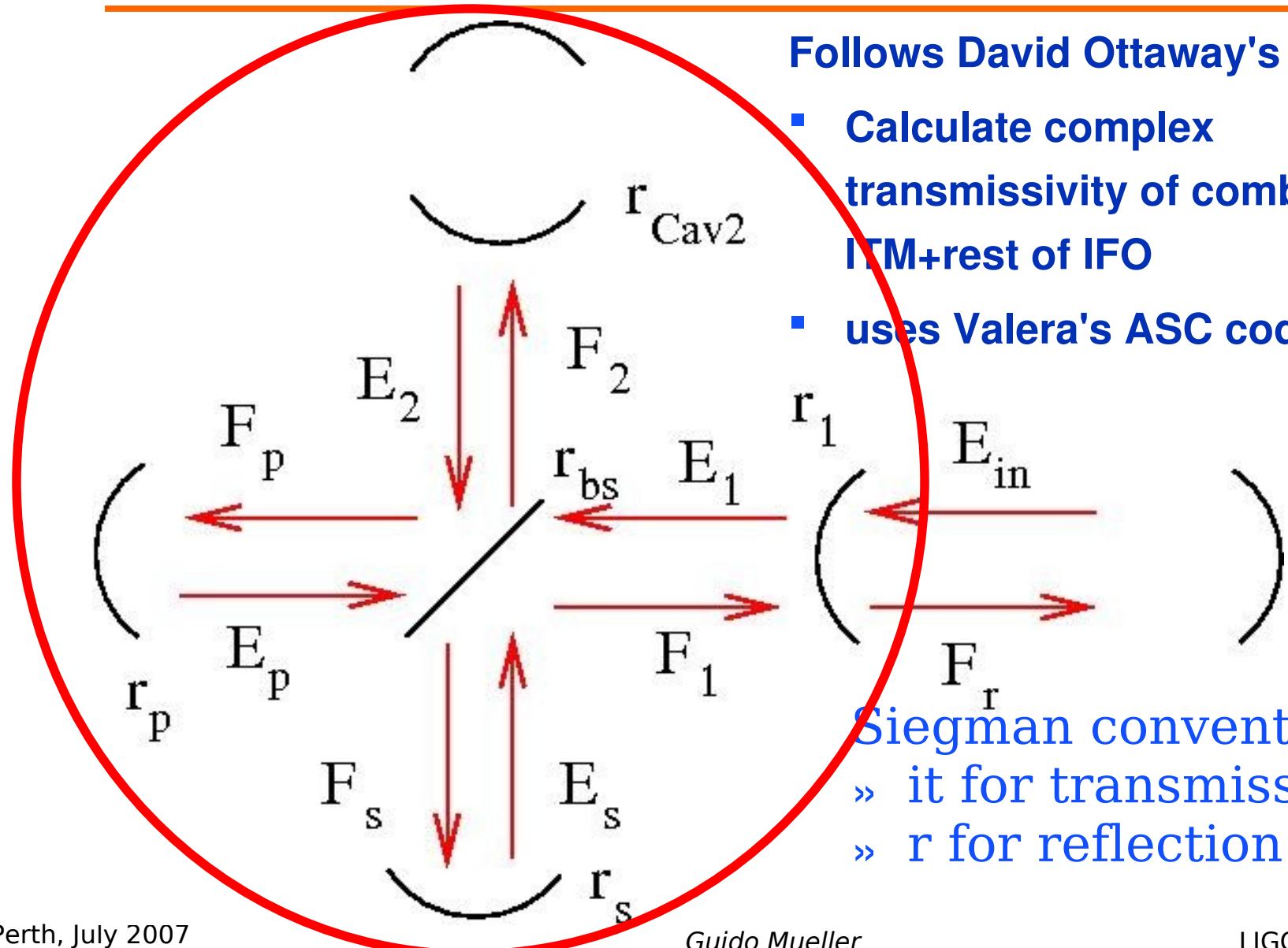


My understanding: PI limited to Hermite-Gauss modes up to  $n$  and  $m < 5$   
 Higher order modes have Diffraction losses  $>$  ITM transmission



Goal would be:

- Compare optical gain of dual recycled interferometer with optical gain in single cavity
  - » Will depend on round trip phase shift of higher order modes in recycling cavities (PI-mode recycling (bad) or extraction (good))
  - » Try to find Gouy phases which reduce  $Q_{\text{opt}}$
- Optimization is currently done
  - » for Alignment sensing (try to get the 10-modes out of the arm cavities to the WFS)
  - » and to suppress Bulls eye (HG 20, HG 02) modes in short Michelson IFO (reduce scatter losses in signal sideband (Yi Pan paper))



Follows David Ottaway's idea:

- Calculate complex transmissivity of combined ITM+rest of IFO
- uses Valera's ASC code

Siegman convention:  
 » it for transmission  
 » r for reflection



Rest



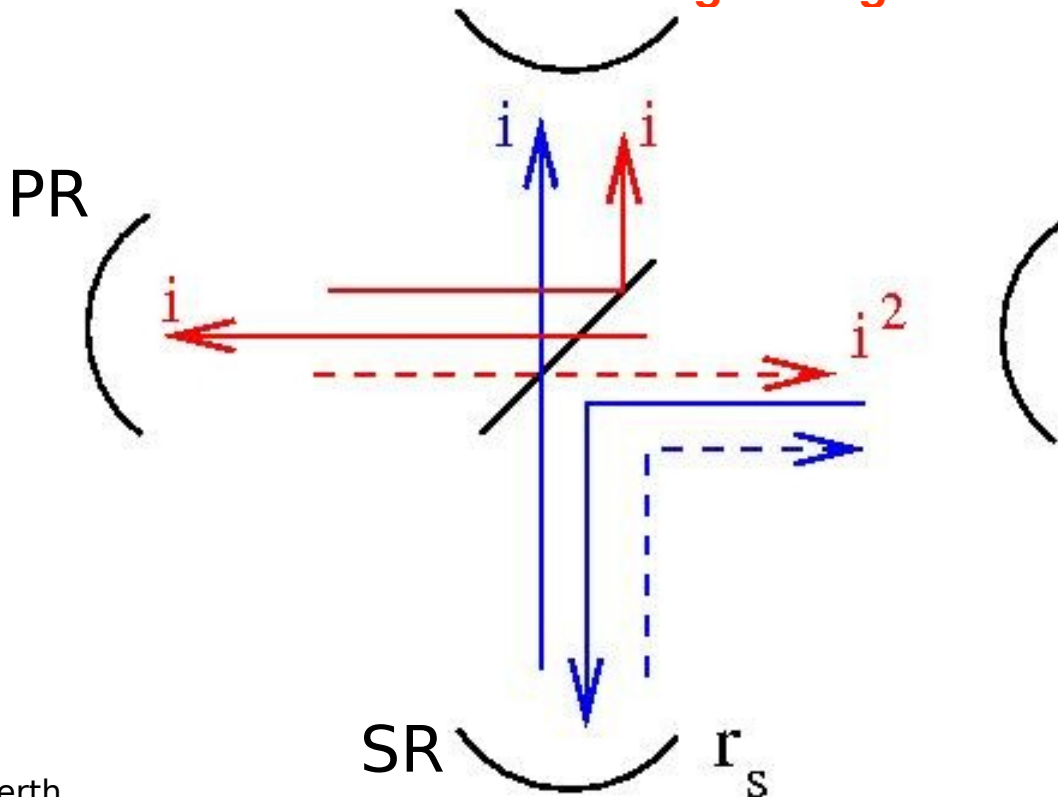
ITM



- Strongly undercoupled cavity
- ( $R_{\text{ITM}} = 0.995$ ,  $R_{\text{rest}} < 0.94$ )
- Roundtrip phase shift:
  - $\pi$ : Power recycling  $\rightarrow$  PI mode will build up
  - $2\pi$ : Signal extraction  $\rightarrow$  PI mode will be damped
- Roundtrip phase depends on Gouy phases and positions of all other mirrors



- Start discussion with following configuration:
  - » symmetric BS:  $t_{BS} = r_{BS}$ , no Gouy phases on recycling arms
  - » PR mirror is power recycling the carrier
  - » SR mirror is extracting the signal mode (RSE configuration)

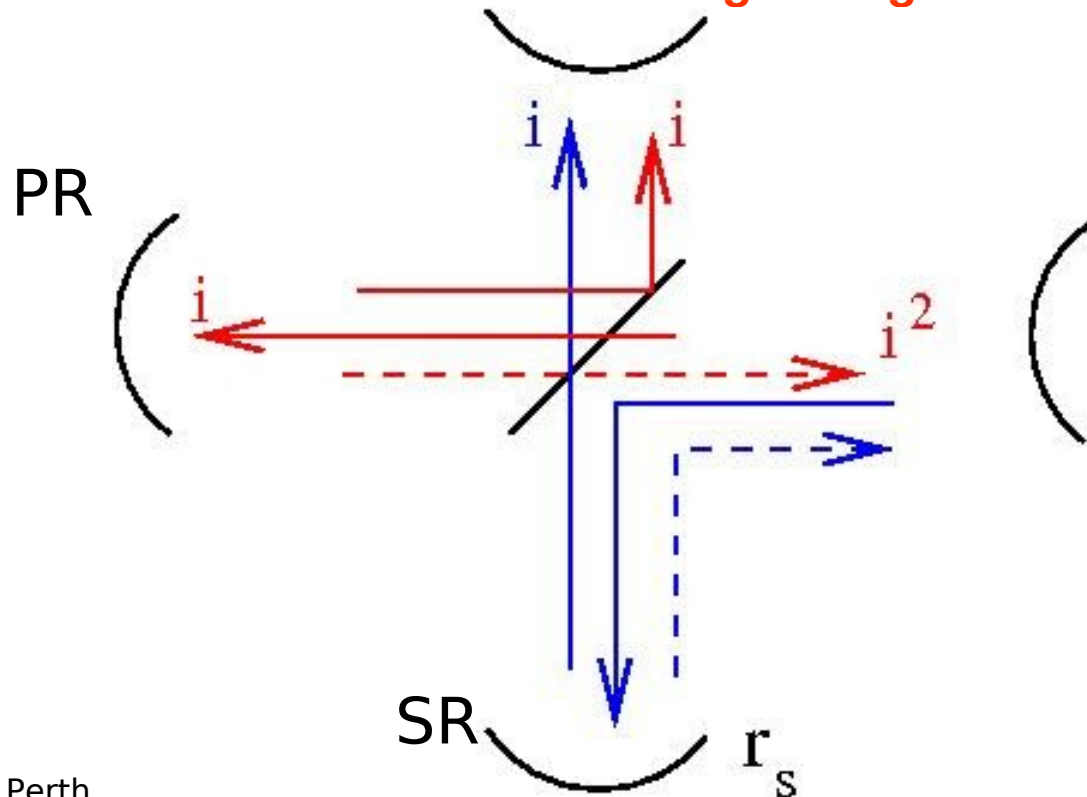


Power recycling:  
 The  $i^2$  ensures  
 destructive interference  
 with cavity leakage  
 at ITM

- ☞ Power recycl.
- † Bad for PI modes.



- Start discussion with following configuration:
  - » symmetric BS:  $t_{BS} = r_{BS}$ , no Gouy phases on recycling arms
  - » PR mirror is power recycling the carrier
  - » SR mirror is extracting the signal mode (RSE configuration)



Signal recycling:  
the **real** SR field ensures  
construct. interference  
with cavity leakage  
at ITM

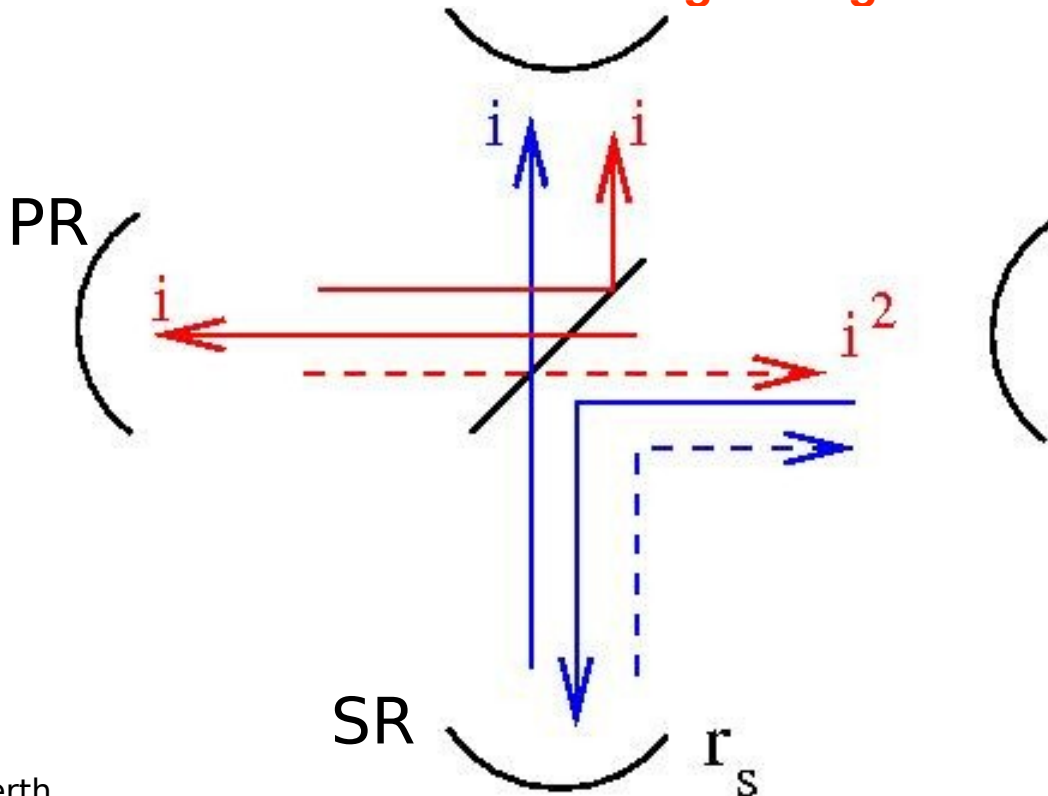
👉 RSE

👌👌 Very good for us





- Start discussion with following configuration:
  - » symmetric BS:  $t_{BS} = r_{BS}$ , no Gouy phases on recycling arms
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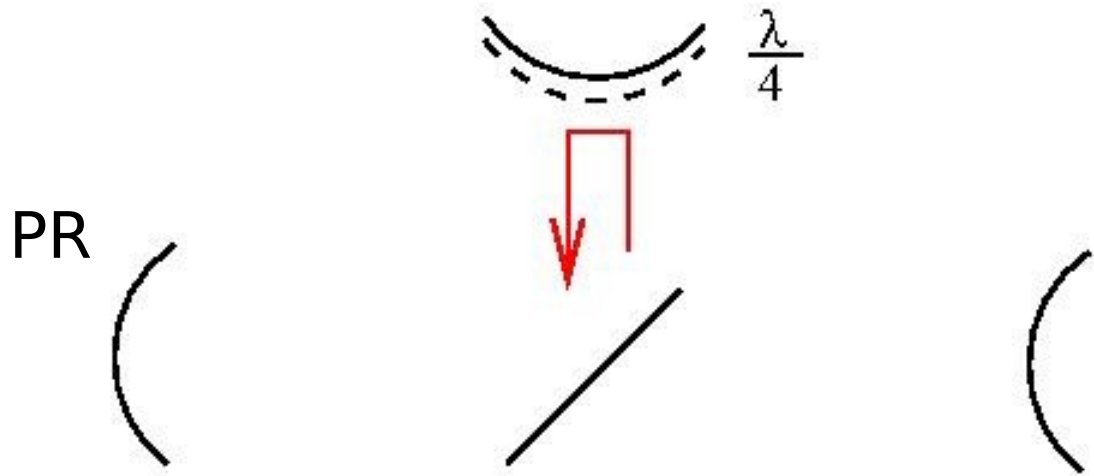
Recycling arm  
Michelson IFO  
would be bright!

☞ PI-mode travels  
100% to 2<sup>nd</sup> Cavity

OK????



2<sup>nd</sup> Cavity moved by  $\lambda/4$  compared to  
1<sup>st</sup> cavity to have MI dark (compensates the  $(it)^2$  in BS)



- 2<sup>nd</sup> Cavity off-resonant
  - ☞ Phase shift:  $\pi$

SR 

- 2<sup>nd</sup> Cavity on-resonance
  - ☞ Phase shift:  $2\pi$

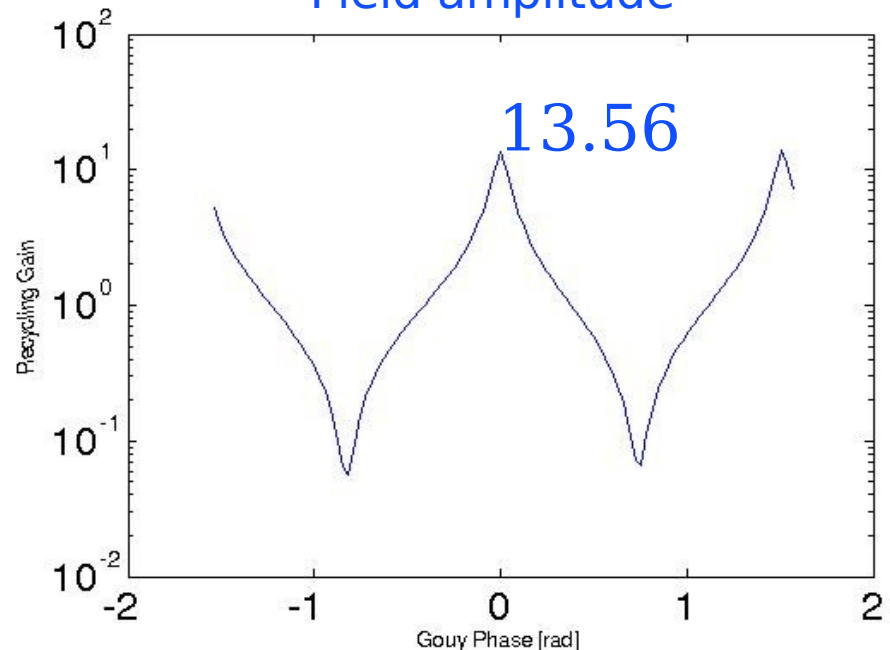


## 1<sup>st</sup> Case:

- 2<sup>nd</sup> Cavity resonant with PI mode (unlikely):  
 Light reflects back into short recycling cavity MI  
 Picks up another 90deg phase shift

- ☞ Round trip phase shift is 180deg ☞ signal recycling (very bad)
- » Add now identical Gouy phases to recycling cavities
  - recycling cavity MI stays bright
  - move from signal recycling to signal extraction

relative recycling gain:  
Field amplitude



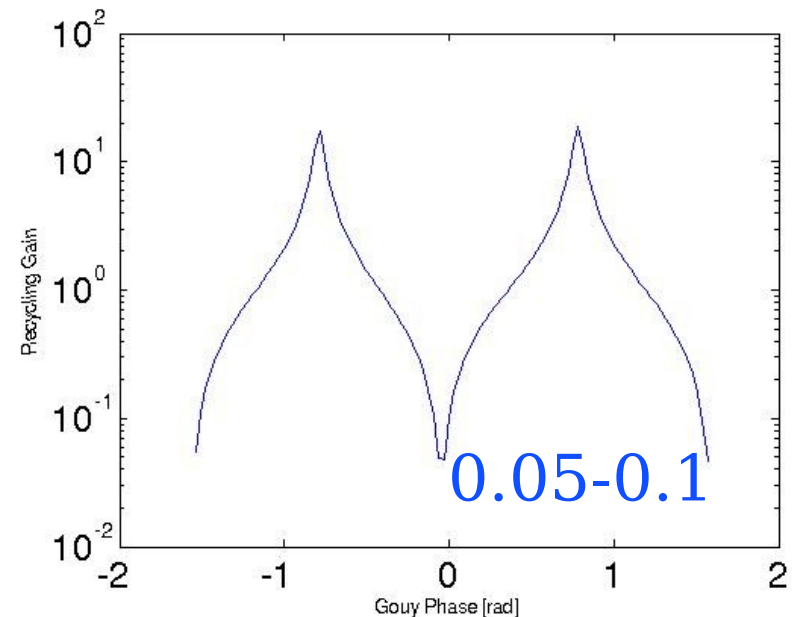


## 2<sup>nd</sup> Case:

- 2<sup>nd</sup> Cavity non-resonant with PI mode (likely):
  - Picks up 180deg phase shift at 2<sup>nd</sup> Cavity
  - Light reflects back into short recycling cavity MI
  - Picks up another 90deg phase shift

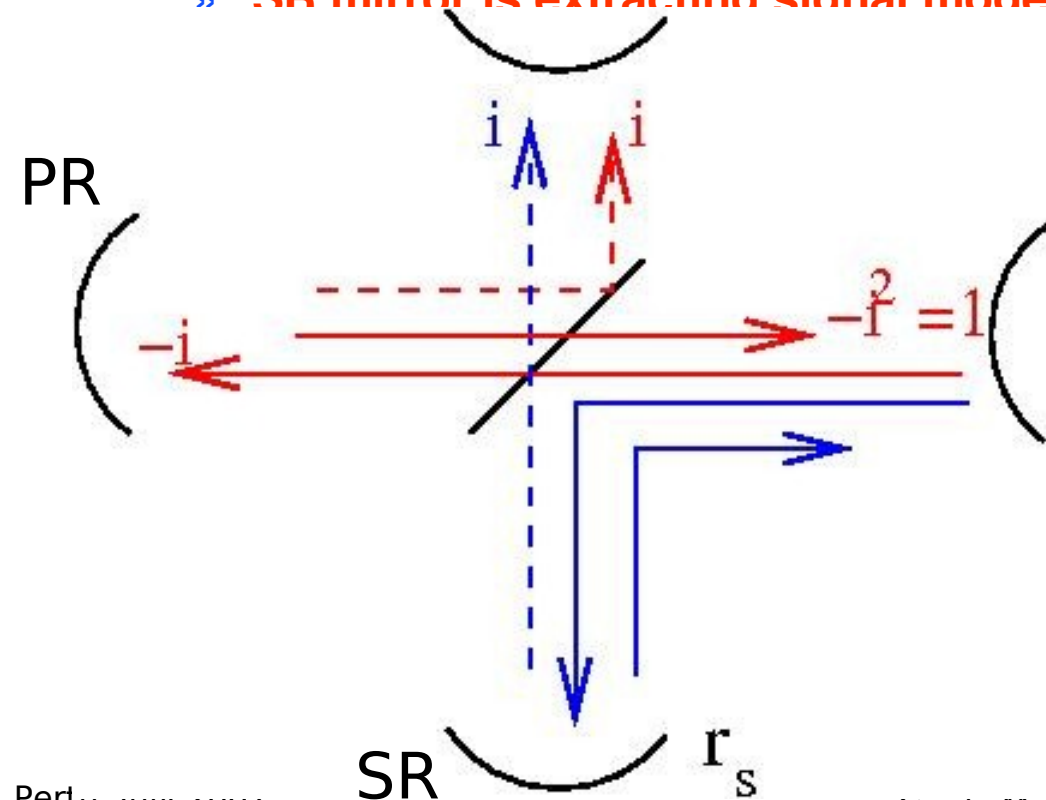
- ☞ Round trip phase shift is 360deg ☞ signal extraction (very good)
- » Add now identical Gouy phases to recycling cavities
  - recycling cavity MI stays bright
  - move from signal extraction to signal recycling

relative recycling gain:  
Field amplitude





- Currently I optimized for alignment sensing (detect the 10-mode) keeping the signal losses in mind (20-mode):
  - » PR recycling cavity would have  $\Psi_G \sim \pi/2$  (to extract the 10-mode)
  - » SR mirror is extracting signal mode (RSE configuration)



Recycling arm  
Michelson IFO  
is dark for odd modes!

☞ 2<sup>nd</sup> Cavity doesn't  
matter anymore

Note:  
Nothing has changed  
for even modes!

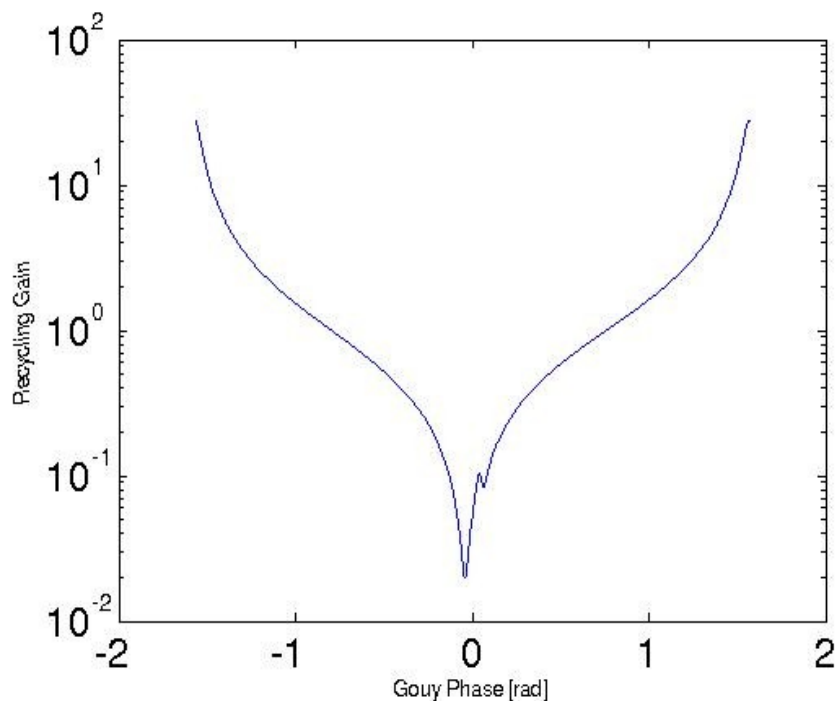


Only new case:

- ➡ **Round trip phase shift is 360deg** ➡ **signal extraction (very good)**
- » **Add now identical Gouy phases to recycling cavities**
  - recycling cavity MI stays dark
  - move from signal extraction to signal recycling

(Structure in center caused by SR-detuning)

relative recycling gain:  
Field amplitude





- One possible design has the following configuration:
  - » Power recycling cavity will have  $\psi_G \sim \pi/2 + 0.1$
  - » Signal recycling cavity will have  $\psi_G \sim 0.1$
- **Odd Modes:**
  - » relative recycling gain  $< 1$  for  $n+m < 8$
- **Even Modes (2<sup>nd</sup> Cavity off-resonant):**
  - » relative recycling gain  $< 1$  for  $n+m < 4$
- **Even Modes (2<sup>nd</sup> Cavity on-resonance, unlikely):**
  - » relative recycling gain  $> 1$  for  $n+m < 4$



- One possible design has the following configuration:
  - » Power recycling cavity will have  $\psi_G \sim \pi/2 + 0.1$
  - » Signal recycling cavity will have  $\psi_G \sim 0.1$
- One problem:
  - » Optimum design for 20-mode suppression is  $\psi_G \sim \pi/4$  rad (Yi Pan's paper)

Ongoing discussion ...