Computing Network Configuration Merit

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Introduction

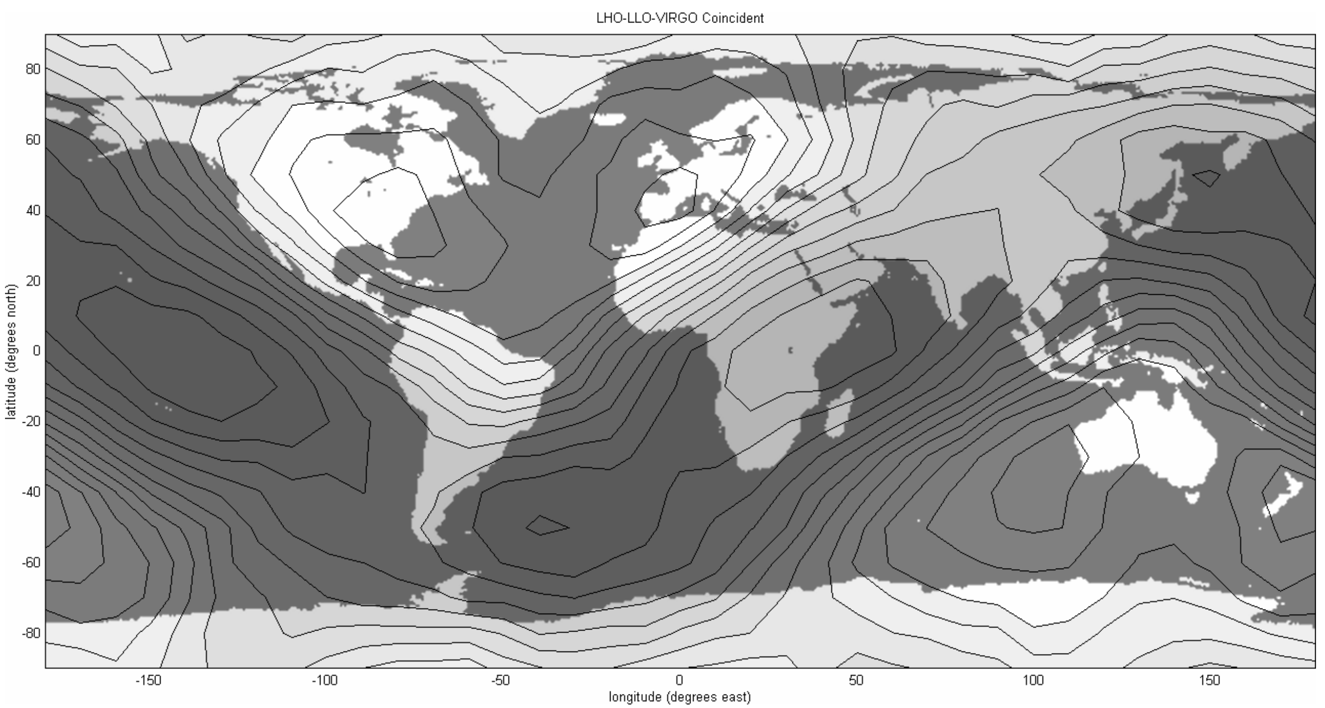
- Gravitational wave astronomy will require the use of co-operative networks of multiple interferometers.
- We naturally wish to determine not only how to optimally analyze data from a given network, but also how to optimally configure the system as a whole.
- We present a formalism enabling the comparison of the quality of given networks under certain criteria.
- This is achieved by the definition of physically significant *figures of merit* for any particular configuration of a global network and analysis system.
- We give particular consideration to the role the proposed Australian-International Gravitational wave Observatory (AIGO) will play in a global network.

Overview

- Figure of merit formalism.
- Binary inspiral detection rate as a figure of merit.
  - Maximum likelihood formulation of network analysis algorithms.
    - Approximations and assumptions for computational amenability.
  - Seeing distance, volume and event rate.
  - Generally applicable figure of merit.
  - Application to optimal augmentation of an existing network.
    - Geographical maps of augmentation site merit.
      - Determination of optimal sites, including AIGO.
    - Interpretation of quantitative and qualitative behavior of analyses.
- Future directions and figures of merit under development.
  - Detectable fraction of galactic pulsars as a figure of merit, with Finn.

Recent Results

- 4th Edoardo Amaldi Conference
- CQG **19** (2002) 1465-1470



Figures of Merit

- Compare the quality of networks α and β by assigning them merits $f(\alpha)$ and $f(\beta)$.
  - α 'better than' β if and only if $f(\alpha) > f(\beta)$.
- A *figure of merit* is a function f assigning a real value to each member of a set of networks $\mathcal{N}$.
  - $f : \mathcal{N} \to \mathbb{R}$

Binary Inspiral Detection Rate

- A figure of merit that is strictly increasing with the rate of detections produced, by a population of binary inspiral events, in any network of detectors, using a particular analysis strategy.
  - It is important that the figure of merit also be amenable to computation.

Binary Inspiral Signal

- The response of a single ideal detector to a binary inspiral is:
  - $$m(t) = \frac{\alpha(t)}{r}\left[(1+\cos^2 i)\cos\beta(t)F_+ + 2\cos i \sin\beta(t)F_\times\right]$$
  - Relative orientations are encoded by antenna patterns F_+ and $F_\times$. r and i are source distance and orbital inclination. $\alpha(t)$ and $\beta(t)$ depend on other source properties (such m_1 and m_2).

Network Analysis Algorithms

- Coincident and coherent network analyses can be similarly implemented as likelihood tests.
  - L. S. Finn, Phys Rev D **63** 102001 (2001)
  - We implement the simplest forms of each test.
    - The performance of both tests could be improved at the cost of added complexity.
    - Coincident veto strategy will be unimportant as we will be concerned only with false dismissals.

Maximum Likelihood Tests

- The likelihood Λ that a signal response $m(t)$ is present in the output $g(t)$ of a detector is:
  - $\ln \Lambda(g|m) = \langle 2g - m, m \rangle$
- The maximum likelihood Λ_{max} is the maximum over a set of signal responses $\mathcal{M}$:
  - $\Lambda_{\max}(g|\mathcal{M}) = \max_{m \in \mathcal{M}} \Lambda(g|m)$
- Detection occurs when a threshold is exceeded:
  - $\Lambda_{\max} > \Lambda$

Coincident Algorithm

- A coincident analysis makes a detection when each detector in a network crosses a threshold:
  - $\Lambda_{\max}(g_i|\mathcal{M}) > \Lambda_i$
  - If each detector in the network is identical and hence has same threshold:
  - $\min_i \Lambda_{\max}(g_i|\mathcal{M}) > \Lambda_{\text{coincident}}$
  - Further vetoes may be applied.

Coherent Test

- A coherent analysis makes a detection when the network as a whole crosses a threshold:
  - $\Lambda_{\max}(\mathbf{g}|\mathcal{M}) > \Lambda_{\mathrm{coherent}}$
  - If the noise is uncorrelated between detectors the network's likelihood is separable:
  - $\max_{m \in \mathcal{M}} \prod_i \Lambda(g_i|m) > \Lambda_{\mathrm{coherent}}$
  - A. Pai *et al*, gr-qc/0110041

False Dismissal

- We are concerned with the false dismissal of events from the population.
  - The selection of thresholds reduces false alarms to an acceptable (insignificant) rate.
- We are then interested in the case when the detector output consists of a signal and additive noise.
  - $g(t) = m(t) + n(t)$

Approximation

- When a signal *m* is present and detected:
  - $\Lambda_{\max}(m+n|\mathcal{M}) > \Lambda_0$
- Approximate mean using the real signal:
  - $$\begin{aligned} \overline{\ln \Lambda_{\max}(m+n|\mathcal{M})} &\approx \overline{\ln \Lambda(m+n|m)} \\ &= \overline{\langle m+2n, m\rangle} \\ &= \langle m, m\rangle \end{aligned}$$

Approximated Tests

- For a coincident analysis:
  - $$\min_i \overline{\ln \Lambda_{\max}(m_i + n_i|\mathcal{M})} \approx \min_i \langle m_i, m_i \rangle > \ln \Lambda_{\text{coincident}}$$
- For a coherent analysis:
  - $$\max_{m \in \mathcal{M}} \sum_i \overline{\ln \Lambda(m_i + n_i|m_i)} \approx \sum_i \langle m_i, m_i \rangle > \ln \Lambda_{\text{coherent}}$$

Seeing Distance and Volume

- Consider a particular source from the population, fixed except for distance *r*.
- Determine the maximum detectable distance:
  - $\min_j \langle m_j, m_j \rangle = \ln \Lambda_{\text{coincident}}$
    $r^2 = \frac{\langle \alpha \cos \beta, \alpha \cos \beta \rangle}{\ln \Lambda_{\text{coincident}}} \min_j [(1 + \cos^2 i)(F_+^2)_j + 4 \cos^2 i (F_\times^2)_j]$
  - $\sum_i \langle m_i, m_i \rangle = \ln \Lambda_{\text{coherent}}$
    $r^2 = \frac{\langle \alpha \cos \beta, \alpha \cos \beta \rangle}{\ln \Lambda_{\text{coherent}}} \sum_j [(1 + \cos^2 i)(F_+^2)_j + 4 \cos^2 i (F_\times^2)_j]$
- Average over source orientations and integrate over the sky for detectable volume, and thus event rate.

Simplifications

- The leading constant does not depend on the geographical configuration of the detectors, only on the source waveform (α, β) and the noise statistics (Λ).
  - $$V = \frac{\langle \alpha \cos \beta, \alpha \cos \beta \rangle}{\ln \Lambda_{\mathrm{threshold}}} \int \cdots \mathrm{d}\Omega$$
- We make a general figure of merit by eliminating the leading constant.

Figure of Merit

- The final form of the figures of merit for:
  - Coincident:

$$\int \{\min_j[(1+\cos^2 i)(F_+^2)_j + 4\cos^2 i(F_\times^2)_j]\}^{\frac{3}{2}}\,\mathrm{d}\Omega$$

  - Coherent:

$$\int \{\sum_j[(1+\cos^2 i)(F_+^2)_j + 4\cos^2 i(F_\times^2)_j]\}^{\frac{3}{2}}\,\mathrm{d}\Omega$$

- Not the detection rate, but linearly proportional to it.
  - Independent of non-geometric source and detector properties, and so valid for any inspiral parameters and any detector noise PSD.

Network Configurations

- Define the set $\mathcal{N}_n$ of networks of *n* detectors each identical up to location and orientation.
  - $\mathcal{N}_n = \{[(\theta_1, \phi_1, \psi_1), \ldots, (\theta_n, \phi_n, \psi_n)]\}$
- Each detector *i* is completely defined by its:
  - Latitude θ_i .
  - Longitude ϕ_i .
  - Orientation ψ_i .

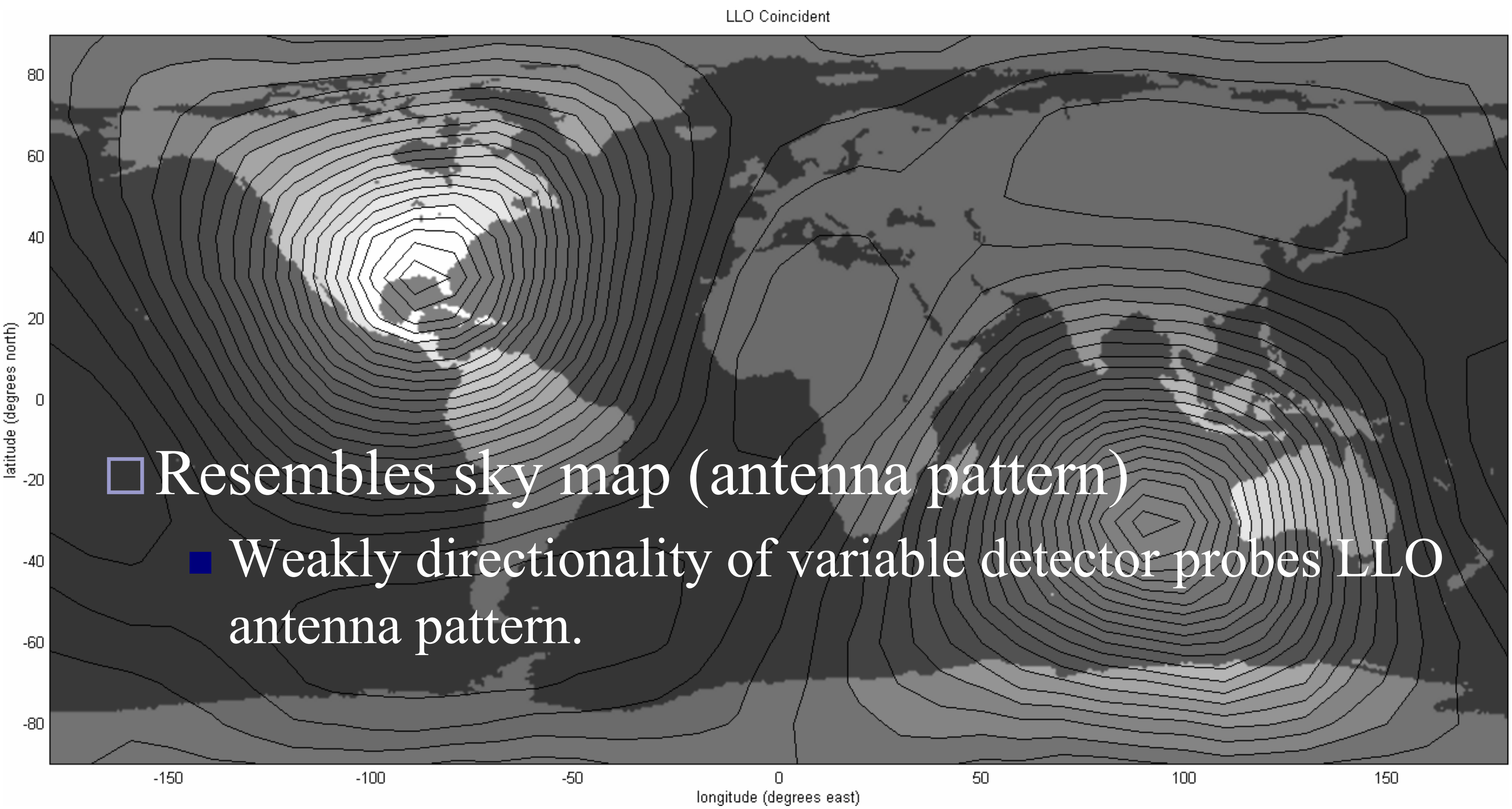
Configurations of Interest

- Consider how to best augment an existing network of $(n-1)$ identical detectors with an nth identical detector for inspiral detection.
  - Fix $(n-1)$ detectors, vary nth detector.
    - Forms a 3-dimensional subset of $\mathcal{N}_n$.
  - Note that merit is insensitive to orientation ψ_n which we fix at an arbitrary value.
    - Forms a 2-dimensional subset of $\mathcal{N}_n$.

Maps of Augmentation Site Merit

- We then compute the figures of merit for points in the two dimensional subset parameterized by the latitude and longitude of the *n*th detector.
  - This set has the straightforward interpretation as a geographical map of the merit of a particular site for augmenting an existing network.
- Integrals evaluated by Monte Carlo methods using the APAC National Facility supercomputer.

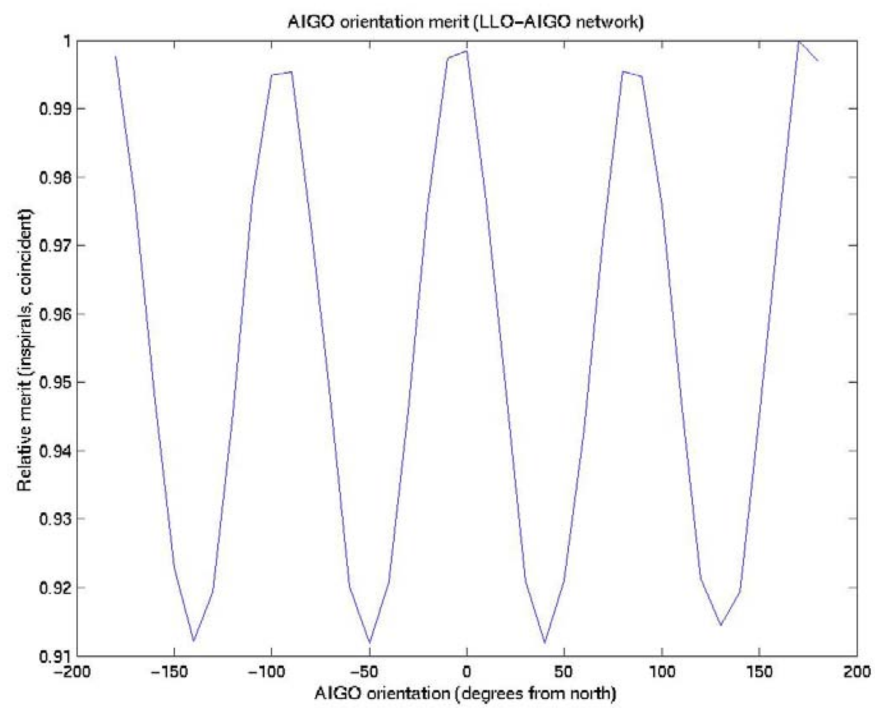
Coincident: LLO



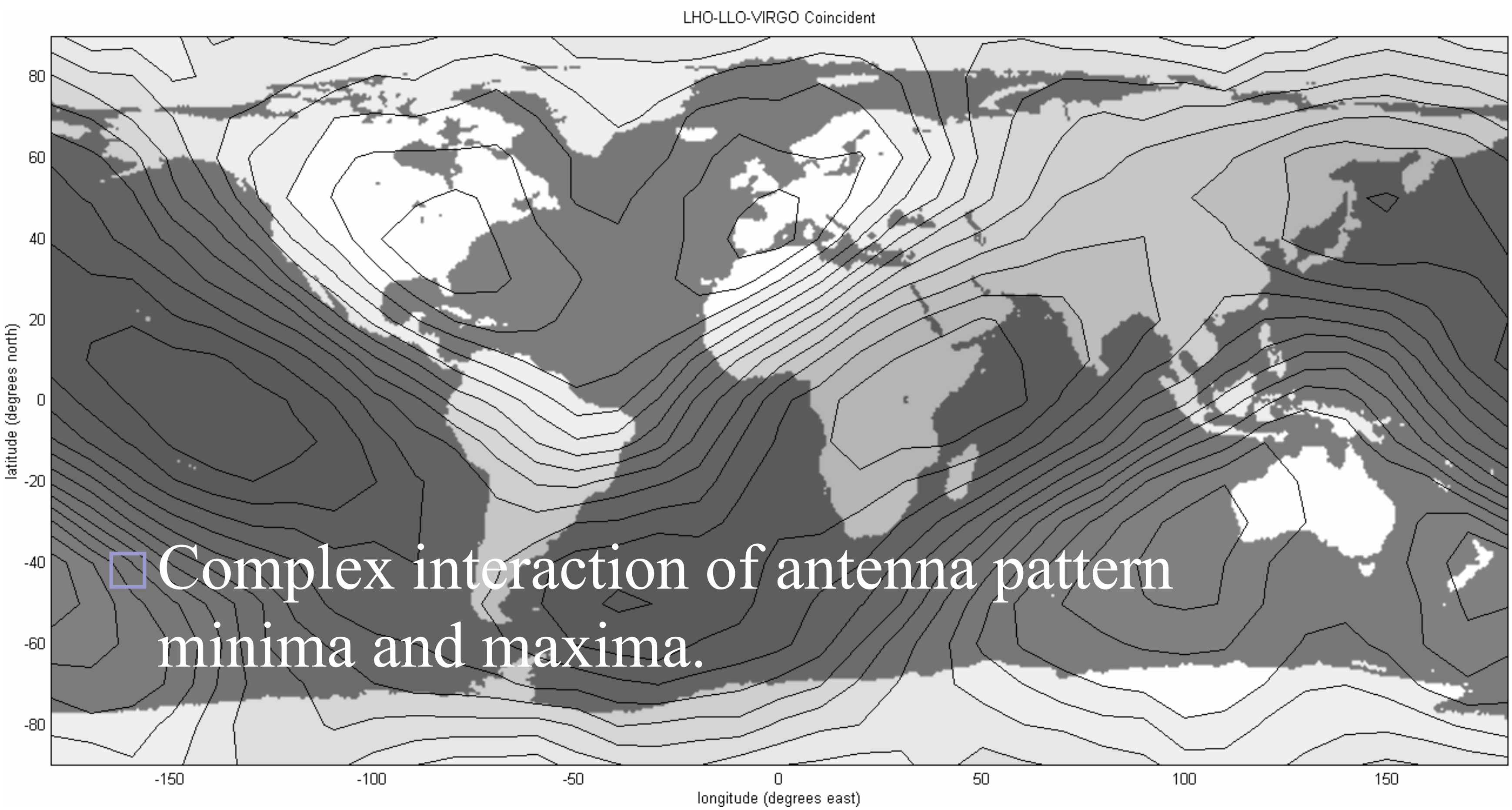
- Resembles sky map (antenna pattern)
  - Weakly directionality of variable detector probes LLO antenna pattern.

Weak Orientation Dependence

- Rotating a detector at the AIGO site varies the merit in the preceding case by < 10%, compared with a 60 % variation from varying the latitude and longitude.
- Other circumstances produce even smaller variations.

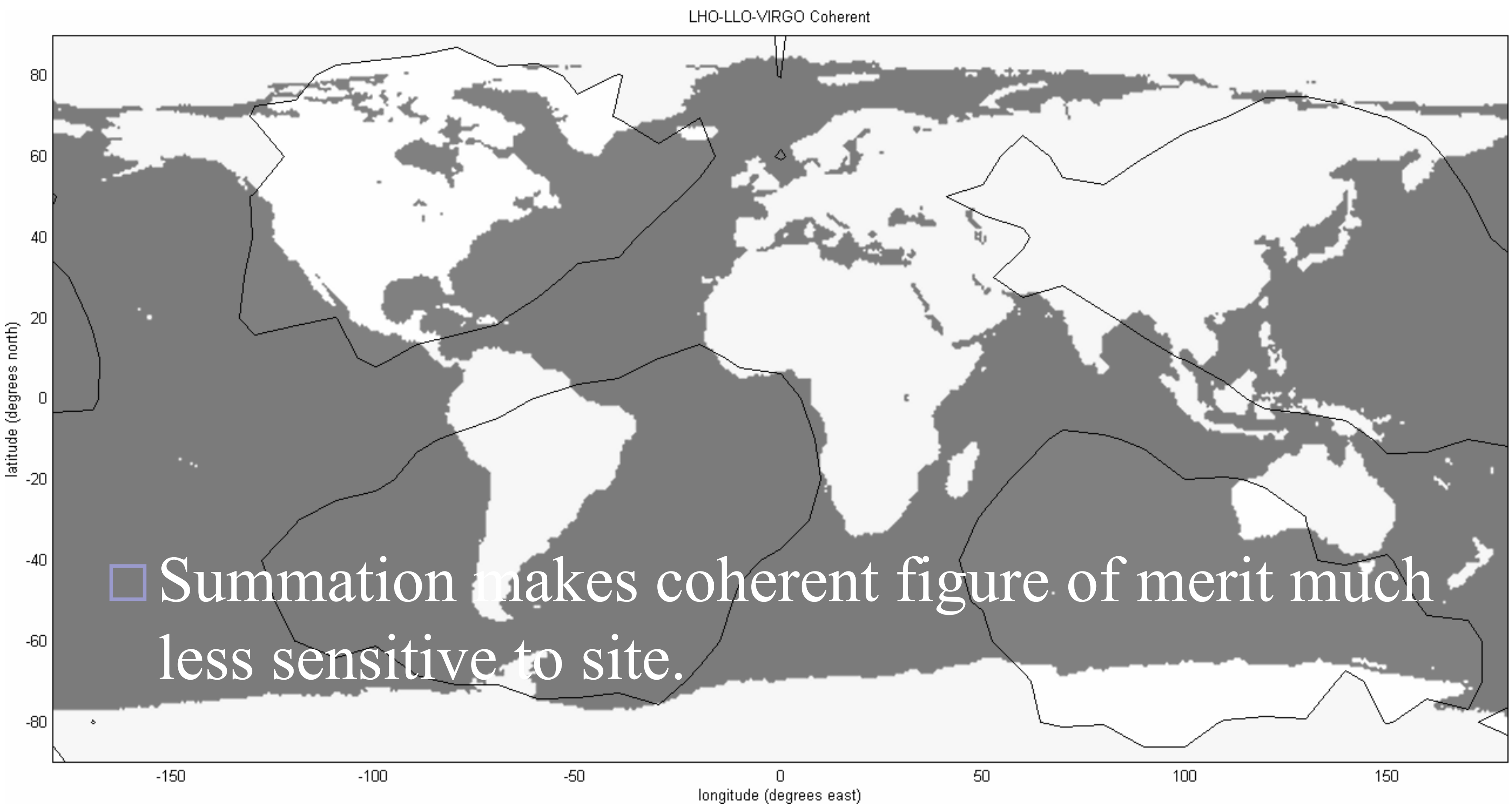


Coincident: LLO + LHO-4k + VIRGO



- Complex interaction of antenna pattern minima and maxima.

Coherent: LLO + LHO-4K + VIRGO



- Summation makes coherent figure of merit much less sensitive to site.

Interpretation

- Results:
  - Coincident analysis sensitive to detector sites.
    - AIGO well placed.
    - N. Arnaud *et al*, gr-qc/0107081
  - Coherent analysis insensitive to detector sites.
  - Realistic networks will be closer to optimal for a coherent analysis.
- Limitations:
  - We can only compare networks with the same number of detectors and the same analysis method.
    - Cannot say if coherent or coincident would produce a higher absolute rate.
    - Cannot say how beneficial adding a detector is.

Future Directions

- We are producing new figures of merit to answer other questions.
  - Improving likelihood approximation to extend binary inspiral detection rate figure of merit applicability.
  - Angular resolution of a network as a figure of merit.

Summary

- We have developed a general formalism for comparing networks and analysis methods.
- Geographical (mis)configuration is a significant factor affecting the detection of binary inspirals for a coincident network analysis, but much less so for a coherent network analysis.
- We are currently working on implementing new and improved figures of merit.